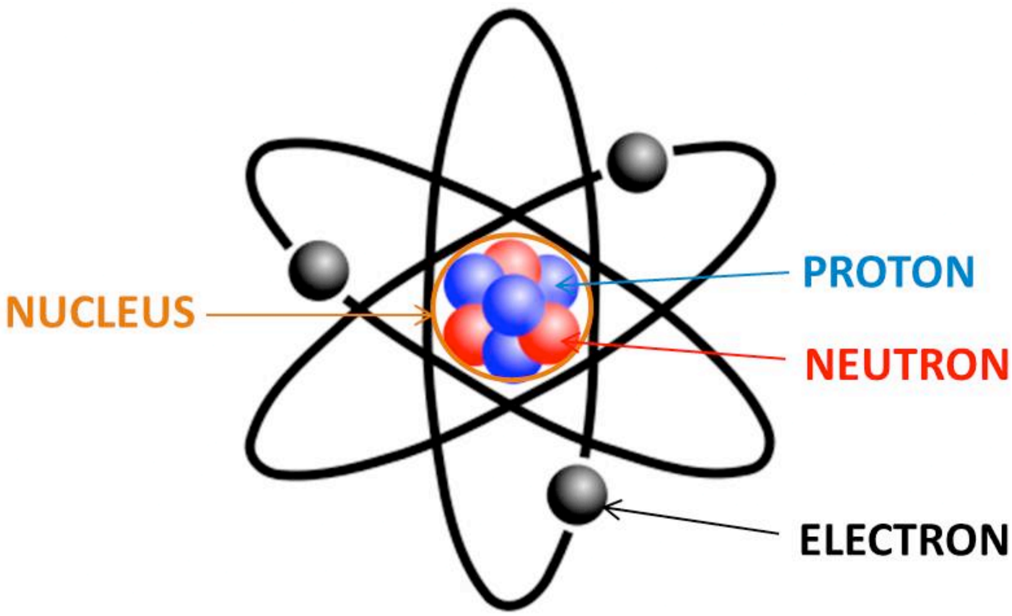
The Atom



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1 The Atom

An atom is the smallest particle of an element that still retains the properties of that element. There are over 100 elements, such as hydrogen, carbon, oxygen, gold, etc. listed in the chemical Periodic Table of Elements shown below.

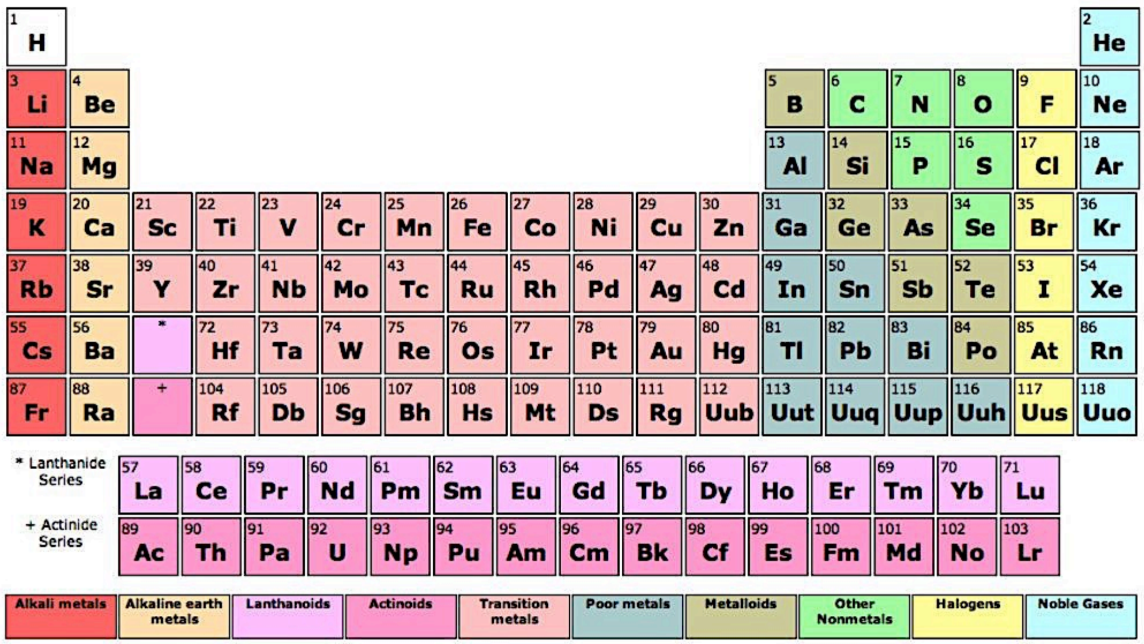


Figure 1 Periodic Table of Elements

2 Discovery of the Atom

The concept of an atom being the smallest particle that something can be divided into was first proposed by the Greek philosophers Leucippus and Democritus around 450 BC. The philosophical question that they were pondering was what would happen if you cut a piece of matter, such as an apple, into smaller and smaller pieces? There are two possible answers to that question:

1. A substance could be divided into ever smaller pieces without limit

2. There is a limit beyond which a substance can no longer be divide.

Democritus reasoned that there was in fact a limit beyond which a substance could no longer be subdivided. Democritus named this smallest particle an atom. Democritus further reasoned that there were only a few types of atoms and that all of matter was composed of various combinations of these atoms. This was very advanced reasoning since at the time it was believed that all types of matter were formed from four basic elements: earth, air, fire, and water.

Aristotle, who lived about 100 years after Democritus, was the most influential philosopher of the early Greek era. Aristotle thought Democritus's idea of atoms was ridiculous. Unfortunately, Aristotle's views were accepted for more than 2000 years. During that time, Democritus's concept of the atom was largely forgotten.

2.1 Dalton Revives Early Ideas About Atoms

Around 1800, a British chemist named John Dalton (1766 – 1844) revived Democritus's early ideas about the atom. From his study of gas pressure Dalton concluded that gases must consist of tiny particles in constant motion.

Figure 2 John Dalton (source: britannica.com)

Through his study of compounds Dalton discovered that a compound always consisted of the same elements in the same ratio. A different compound contained a different combination of elements, again in a specific ratio. This lead Dalton to concluded that elements were made of tiny particles that could combine in various ways to produce an endless variety of compounds. Based on this work Dalton developed the following theory of atoms:

1. All substances are made of atoms.

2. Atoms are the smallest particles of matter. They cannot be divided into smaller particles. They also cannot be created or destroyed.

3. All atoms of the same element are alike and have the same mass. Atoms of different elements are different and have different masses.

4. Atoms join together to form compounds. A given compound always consists of the same kinds of atoms in the same ratio.

Dalton's theory became widely accepted. The only part of Dalton's theory that turned out to be incorrect was his idea that atoms could not be divided into smaller particles.

Today we know that an atom can be subdivided. We know that every atom is composed of a dense nucleus consisting of protons and neutrons surrounded by a cloud of electrons as illustrated in Figure 3.

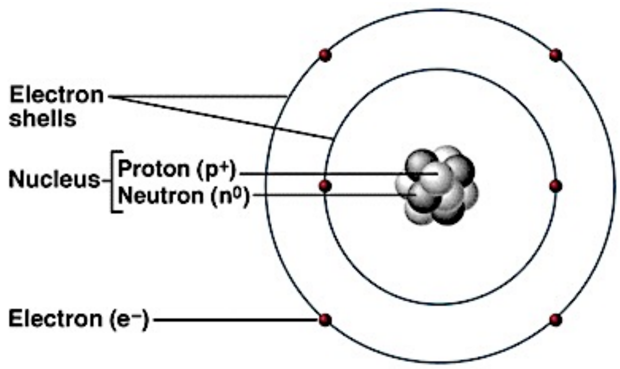


Figure 3 Simple diagram of an atom (source: http://igcsetuition.blogspot.com)

As with most complex systems, the atomic structure was discovered piecemeal. The first direct evidence of atoms occurred in 1827. In 1897 the electron was discovered, the nucleus in 1911, the proton in 1919, and finally the neutron in 1932.

2.2 Direct Evidence of Atoms

Robert Brown (1773 – 1858), a Scottish botanist, is credited with the first direct evidence of atoms. In 1827 Brown used a microscope to observe tiny pollen grains suspended in still water. He noticed that the pollen grains, for some unknown reason, moved about in complex irregular paths. The motion turned out to be caused by random thermal motion of the fluid's molecules colliding with the pollen grains. This movement is now called Brownian motion. Fluctuations in the number of molecules striking a pollen grain cause it to move first in one direction then in another. Although the molecules could not be directly observed, their effects on the pollen grains was.

Figure 4 Robert Brown (source: https://dailyasianage.com)

2.3 Discovery of Electrons

In 1857 German physicist and glassblower Heinrich Geissler invented the gas discharge tube (initially called the Geissler Tube). It consisted of a sealed partially evacuated glass cylinder with a metal electrode at each end similar to that shown in Figure 5.

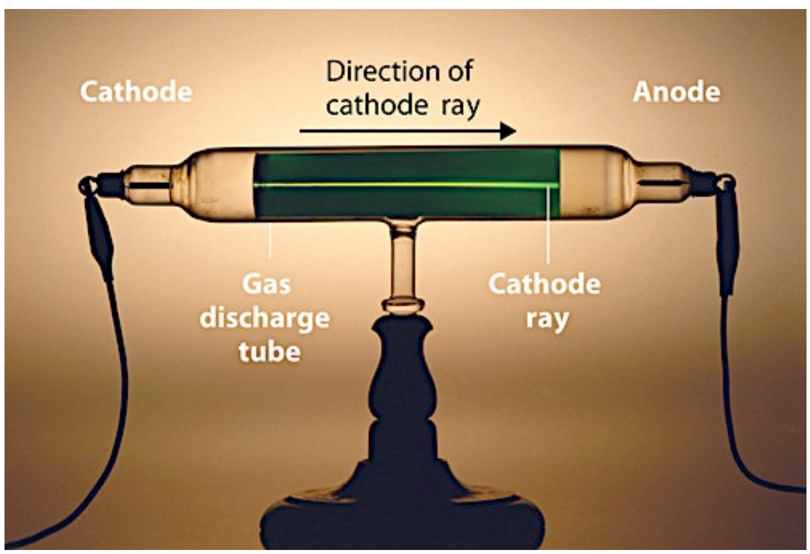


Figure 5 Gas Discharge Tube producing a cathode rays (source: CC BY-SA-NC)

The tube was filled with a rarified gas such as neon, argon, or air.

An electrical current flowed through the tube when a high voltage was applied between the electrodes. In Figure 5 the negatively charged electrode is the cathode while the anode is the positive electrode. The current dissociated electrons from gas molecules along the current's path producing ions. Light was produced when the electrons recombined with ions forming a visible ray that spread out and faded as it moved away from the cathode. The ray was called a cathode ray since it emanated from the cathode. The color of the light produced depended upon that type of gas in the tube.

A number of scientists, including Geissler, English scientist William Crookes, and others experimented with the gas discharge tube in an attempt to determine the nature of cathode rays. Crookes discovered that placing a magnetic near the tube caused the cathode ray to be bent in a direction that indicated that the ray was composed of negatively charged particles.

In the late 19th century, physicist J.J. Thomson (1856–1940) improved and expanded on the experiments with gas discharge tubes. The gas discharge tube apparatus that Thomson used in his experiments was similar to that shown in Figure 7.

Figure 6 J. J. Thomson (source: www.firstworldwar.com)

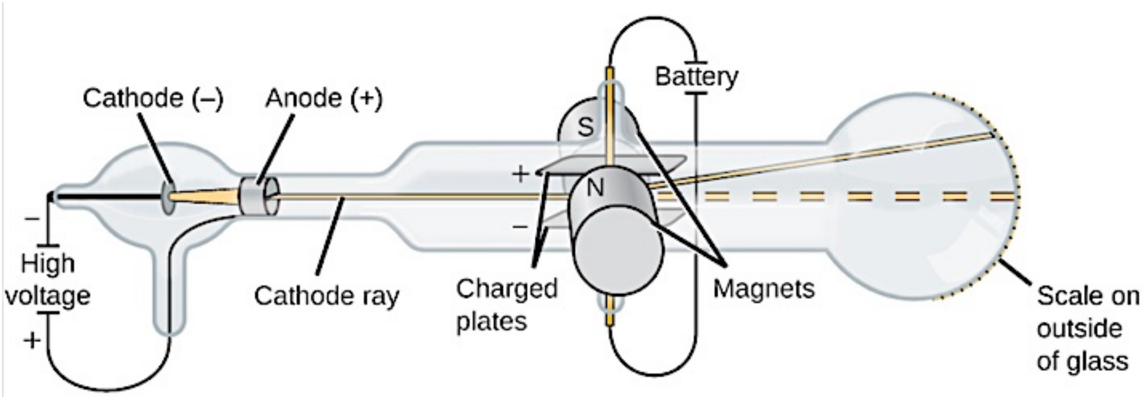


Figure 7 Diagram of J.J. Thomson's cathode ray tube (source: Openstax, CC BY 4.0)

In Thomson's configuration a cathode ray was emitted from the negative cathode element and flowed to the positive anode as in conventional cathode ray tubes. However, Thompson placed a slit in the anode permitting a narrow beam of cathode rays to continue through the anode and eventually striking a phosphorous screen deposited on the inside of the globe forming the far end of the tube. Flashes of light occurred on the screen as the cathode ray particles impacted the phosphorous coating.

Thomson placed two oppositely-charged electric plates, one above and the other below the tube, to test the properties of the cathode ray. The cathode ray was deflected away from the negatively-charged electric plate and towards the positive plate. This verified Crookes' earlier results that the cathode ray was composed of negatively charged particles. In addition, Thomson placed two magnets on either side of the tube as shown in Figure 7. The field from the magnetics also deflected the cathode ray.

Thomson referred to the cathode ray negatively charged particles as corpuscles. This name was later changed to electrons by the Irish physicist George F. Fitzgerald.

Utilizing his apparatus Thomson was able to measure the ratio of an electron's charge (q_e) to its mass (m_e).

$$\frac{q_e}{m_e}$$

This was a necessary step in ultimately determining the actual values for both q_e and m_e

The electric field E perpendicular to the magnetic field B of the magnet produced opposing forces on the electrons. By adjusting the two fields the net force on the cathode ray electrons could be reduced to zero, in which case the velocity v of the electrons became

$$v = \frac{E}{B}$$

In this manner, Thomson was able to determine the velocity of the cathode ray electrons and then moved the beam up and down by adjusting the electric field.

The deflection of the cathode ray was proportional to the electric force on an electron as well as the electron's mass and acceleration (a)

$$F = q_e E = m_e a$$

The acceleration of an electron was then

$$a = \frac{F}{m_e}$$

The value of F was not known, since the charge of an electron q_e was not yet known. Substituting the expression for electric force into the expression for acceleration yielded

$$a = \frac{F}{m_e} = \frac{q_e E}{m_e}$$

By rearranging terms the ratio of electron charge to mass became

$$\frac{q_e}{m_e} = \frac{a}{E}$$

Acceleration was determined by the amount of cathode ray deflection while the electric field E was determined from the voltage and the distance between the parallel electric plates. Thus the ratio

$$\frac{q_e}{m_e}$$

could be determined.

With the velocity

$$v = \frac{E}{B}$$

known, the ratio

$$\frac{q_e}{m_e}$$

could also be determined by the bending of the cathode ray beam resulting from the applied magnetic field since

$$F_{mag} = q_e v B = m_e a$$

Again rearranging terms gives

$$\frac{q_e}{m_e} = \frac{a}{vB}$$

Consistent results were obtained using both electric and magnetic field deflection.

The value obtained for the ratio was

$$\frac{q_e}{m_e} = -1.76 \cdot (10^{11})\ \frac{C}{kg} \qquad (for\ an\ electron)$$

where C is coulombs.

The ratio was a huge number. It implied that the mass of an electron was much, much, smaller than that of any know atom.

Thomson performed a similar experiment on positively charged hydrogen ions. A hydrogen ion (a hydrogen atom with its single electron stripped away) is simply a proton. However, Thomson did not know that at the time.

Thomson discovered that the ratio of a hydrogen ion charge (q_p) to its mass (m_p) was

$$\frac{q_p}{m_p} = 9.58 \cdot (10^{7})\ \frac{C}{kg} \qquad (for\ a\ proton)$$

This is about a 2,000 times less than the electron charge to mass ratio indicating that a hydrogen ion (a proton) was about 2,000 times more massive than an electron. Today we know that the mass of a proton is 1,836 more than the mass of an electron, that is

$$m_p = 1836\ m_e.$$

From his experiments Thomson concluded that:

- The cathode ray is composed of negatively-charged particles.
- The particles must be part of the atom, since the mass of each particle is only $\sim \frac{1}{2{,}000}$ the mass of a hydrogen atom.
- These subatomic particles can be found within atoms of all elements.

Thomson published his results in 1897. His discoveries, particularly the small mass of an electron, were at first controversial. However, his results gradually became accepted by the scientific community.

Discovery of the electron disproved that part of Dalton's atomic theory which assumed that atoms were indivisible. Consequently, an entirely new atomic model was needed to account for electrons.

In 1904 Thomson proposed that an atom consisted of a relatively large positively charged sphere throughout which very small electrons were dispersed, as illustrated in Figure 8. The number of negatively charged electrons was just sufficient to cancel the positive charge so that the atom would be electrically neutral. Thomson's model was based on his experimental results. Namely

- Naturally occurring atoms were electrically neutral
- The mass of a hydrogen atom was nearly 2,000 times greater than an electron
- An electron was negatively charged

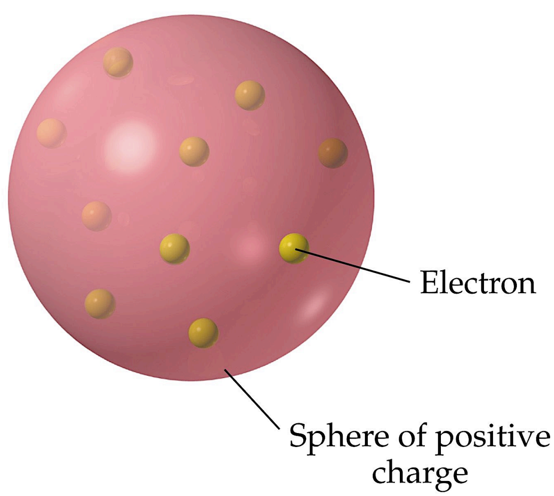


Figure 8 Thomson's Model of the Atom (source: http://atom-history.weebly.com)

Thomson's model of the atom soon became named the "plum pudding model" since the distribution of electrons within the positively charged part of the atom reminded many scientists of raisins, then called "plums," in the common English dessert, plum pudding.

Thomson was awarded the 1906 Nobel Prize in Physics for his pioneering work on the atom.

2.4 The Millikan Oil Drop Experiment

In 1909 American physicist Robert Millikan (1868–1953) conducted an oil drop experiment to determine the charge q_e of an electron. The experiment produced the first accurate direct measurement of an electron's charge, one of the most fundamental constants in nature.

For the experiment, fine drops of oil were sprayed from an atomizer into the upper chamber of the apparatus illustrated in Figure 10 and 11. Some of the oil droplets fell through a pin hole in the center plate (red plate in the diagram) into the lower chamber. X-rays were injected into the lower chamber ionizing the air. Free electrons from the ionized air attached themselves to the oil droplets giving the drops a negative charge. A light source illuminated the falling droplets permitting them to be seen by a microscope installed in the lower chamber wall. A high voltage was placed between the red plate and the lower blue plate creating a uniform electric field E between the two plates. The magnitude of the electric field was equal to

$$E = \frac{V}{d}$$

where V was the voltage applied between the two plates and d the distance between them.

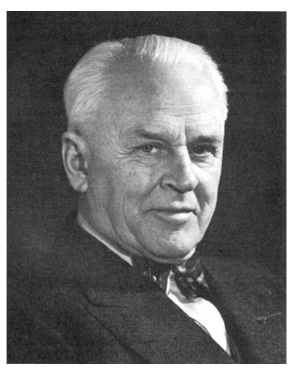


Figure 9 Robert Millikan (source: Wikimedia Commons)

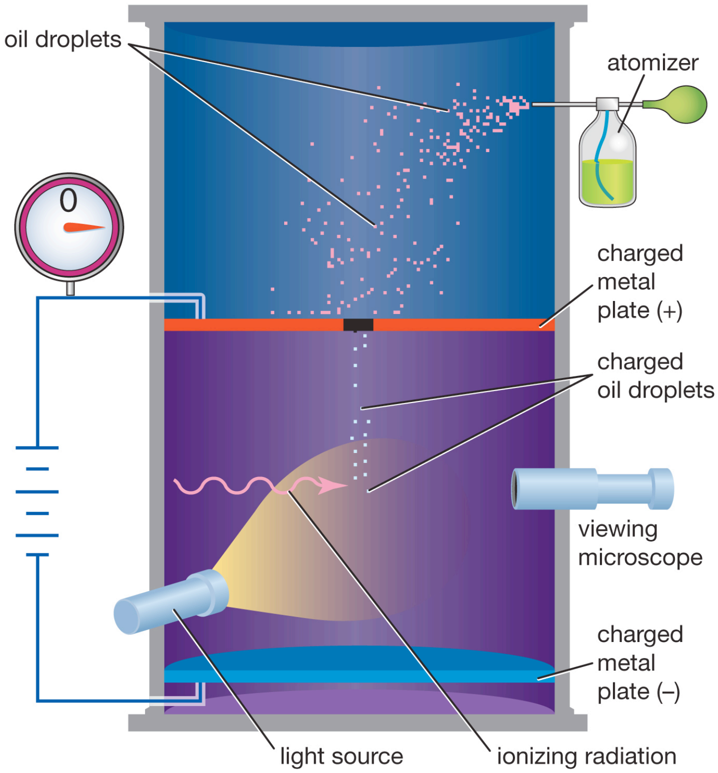


Figure 10 Millikan Oil Drop Experiment

(source: Encyclopedia Britannica)

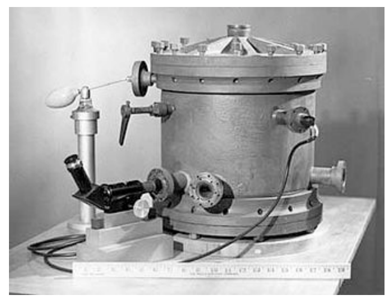


Figure 11 Actual Unit

(source: Wikipedia)

By varying the electric field, the downward force of gravity on a droplet ($m_{drop} \cdot g$) was balanced by the upward force of the electric field ($q_e E$) causing the droplet to be suspended between the two electrical plates. When suspended

$$m_{drop} \cdot g = q_e E$$

where

m_{drop} = mass of the droplet

g = acceleration of gravity

q_e = charge on the droplet

E = electric field produced by the voltage between the upper and lower plates.

The drops were seen in the microscope as points of reflected light. A single suspended drop could be observed for an hour or more by keeping the voltage between the two plates constant.

The drops were too small to directly measure their size and mass. Instead, the mass of a drop was determined by observing how fast it fell through the lower chamber when the voltage was turned off. Oil drops were used instead of water, because oil does not readily evaporate. Consequently, the mass of an oil drop remained relatively constant.

Once the mass of an oil drop was known, the charge of the electron was given by:

$$q = \frac{m_{drop} \cdot g}{E} = \frac{m_{drop} \cdot g \cdot d}{V}$$

By 1913 Millikan had measured the charge of an electron q_e to an accuracy of 1% arriving at a value of -1.60×10^{-19} C, where C is coulomb.

Once the charge of an electron was known, along with the electron charge-to-mass ratio from Thompson's experiments, the mass of an electron could be determined using the following equation:

$$m = \frac{q_e}{q_e/m_e}$$

Substituting in the values for q_e/m_e and q_e gave

$$m_e = \frac{-1.6 \cdot (10^{-19})\ C}{-1.76 \cdot (10^{11})\ C/kg} = 9.11 \times 10^{-31}\ kg$$

that is, the mass of an electron $m_e = 9.11 \times 10^{-31}$ kg . A similar calculation gives the mass of a proton now know to be $m_p = 1.67 \times 10^{-27}$ kg . This mass is nearly identical to the mass of a hydrogen atom.

What Thomson and Millikan had done was to prove that the atom itself was composed of smaller subatomic particles, one being the electron. They also showed that the mass of an electron was only a tiny fraction of the mass of an atom.

Millikan was awarded the 1923 Nobel Prize in Physics for his direct measurement of q_e and for his studies of the photoelectric effect.

2.5 Discovery of the Nucleus

Physicist Ernest Rutherford (1871 – 1937) began his career as a research student for J.J. Thomson at the Cavendish Laboratory, University of Cambridge from 1895 to 1898.

Figure 12 Ernest Rutherford

While at Cambridge he and Thomson discovered that x-rays ionized gases, a property used by Milliken in his oil drop experiment.

While Thomson was studying the charge-mass ratio of electrons, Rutherford focused on the radiation being emitted by radioactive substances. He found that the radiation was more complex than previously thought. There seemed to be two types of radiation involved. One type, that he called alpha (α) radiation was easily stopped by paper and foil. The other type, that he called beta (β) radiation, easily passed through the same foil. In addition to these two types of radiation, a third type of radiation was discovered in 1900 by French chemist Paul Villard when investigating the radiation from radium. Several years later Rutherford suggested that this third type of radiation be called gamma-rays (γ) to be consistence with alpha and beta radiation as the first three letters of the Greek alphabet $(\alpha,\ \beta,\ \gamma)$. The idea stuck.

It was found that $\alpha,\ \beta, and\ \gamma$ rays acted differently when passed through an electric field (Figure 13a) and a magnetic field (Figure 13b). β rays were bent toward the electric field positive plate while α rays were bent in the direction of the negative plate, indicating that β rays were negatively charged while α rays possessed a positive charge. The same result was confirmed by passing $\alpha\ and\ \beta$ rays through a magnetic field. However, the γ rays were not affected by either electric or magnetic fields.

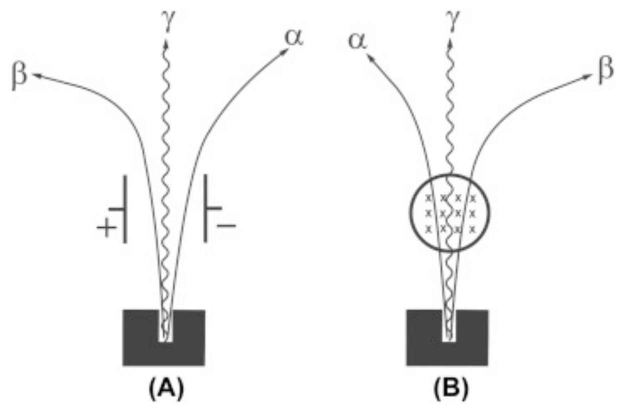


Figure 13 Deflection of $\alpha,\ \beta, and\ \gamma$ Rays (source: www.sciencedirect.com)

After 3 years at the Cavendish laboratory, Rutherford moved in 1898 to McGill University in Montreal at the age of 27 to take the position of professor of physics. At McGill University Rutherford discovered that radioactive atoms that emit either α or β particles disintegrate into different types of lighter weight atoms. Furthermore, he demonstrated that the lighter weight atoms often were radioactive themselves and consequently disintegrated into even lighter atoms. He showed that the chain of disintegration continued until a stable atom was produced. For example, we know today that the decay chain for Radium-226 (Figure 14) is:

Radium-226 (1600 year half life) > Radon-222 + α particle

Radon-222 (3.82 day half life) > Polonium-218 + α particle

Polonium-218 (3.05 minute half life) > Lead-214 + α particle

Lead-214 (26.8 minute half life) > Bismuth-214 + β particle

Bismuth-214 (19.7 minute half life) > Polonium-214 + β particle

Polonium-214 (0.16 millisecond half life) > Lead-210 + α particle

Lead-210 (22 year half life) > Bismuth-210 + β particle

Bismuth-210 (5.0 day half life) > Polonium-210 + β particle

Polonium-210 (138 day half life) > Lead-206 + α particle

Lead-206 is STABLE.

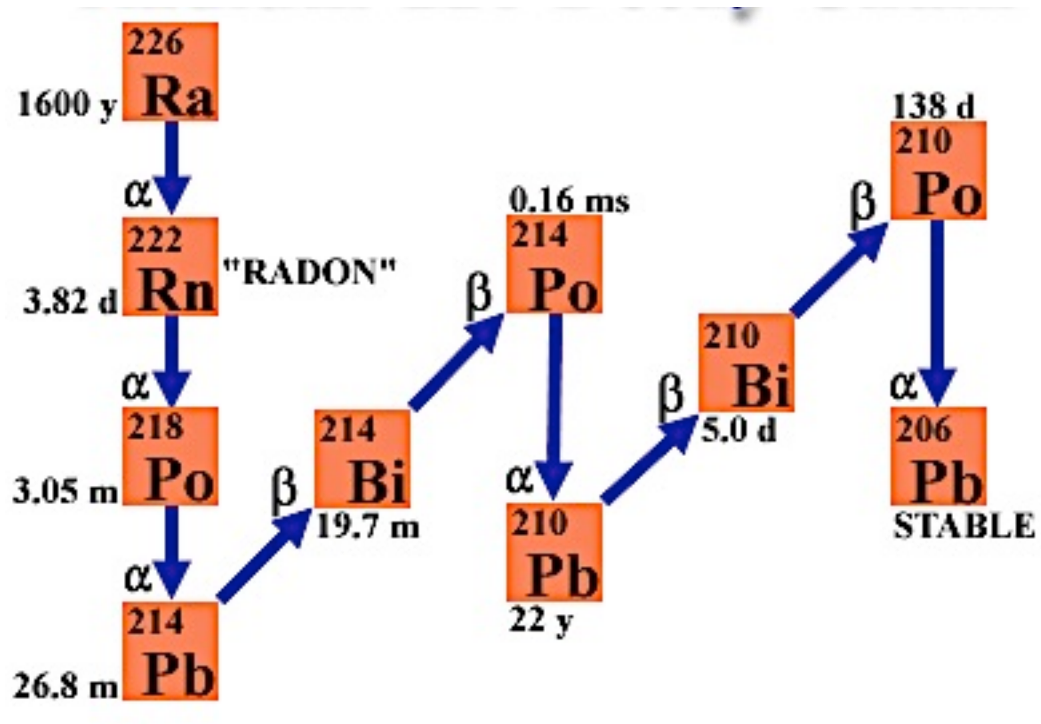


Figure 14 Radium Decay Chain (source: National Institute of Standards and Technology)

Notice that if an α particle (a helium nuclei consisting of 2 protons + 2 neutrons) is ejected from the nucleus of a disintegrating atom, the atomic number (the number of protons in the new atom) is two less than the original atom while its mass (the number of protons + neutrons) is four less. Thus Radium $_{88}Ra^{226}$ disintegrates to Radon $_{86}Rn^{222}$. If a β particle (the electron resulting from the disintegration of a neutron into a proton + electron) is ejected from the nucleus of a disintegrating atom, the atomic number of the new atom (the number of protons) will be one more than the original atom while its mass (the number of protons + neutrons) will be the same as the original atom. For example, the disintegration of lead $_{82}Pb^{214}$ into Bismuth $_{83}Bi^{214}$. At the time Rutherford did not know these details since protons and neutrons had not yet been discovered.

In this work, Rutherford and his coworkers were able to demonstrate that the α particle was actual a helium atom, or more correctly, as later learned, the nucleus of a helium atom.

In 1907 Rutherford moved to England to fill the position of professor of physics at Manchester University. The following year (in 1908) Rutherford was awarded the Nobel Prize in Chemistry for the work that he had done at McGill University in developing his atomic radioactive disintegration theory.

Rutherford had a list of research topics that he wanted to explore at Manchester. One of these was the deflection of α particles as they passed through very thin foil such as aluminum or gold.

One of his new students, Ernest Marsden, needed a research project. So Rutherford suggested that they see if any alpha particles would be deflected at large angles (over 90 degrees) when passing through a thin gold foil. From experience α particles passed straight through thin foils, so Rutherford did not expect to see any large angle deflections, but the question had to be investigated.

The experiment he devised is shown in Figure 15. Radioactive radium was used as a source of α particles to bombard a thin foil of gold. The radium was enclosed in a lead box. A small hole in the box formed a beam of α particles aimed at the gold foil. A screen of zinc sulfide was placed around and behind the foil to detect any scattering of α particles. The experiment had to be conducted in the dark so that microscopic flashes of light could be seen each time α particles struck the screen.

As illustrated in Figure 15, most of the α particles passed through the gold foil with little or no deflection striking the screen directly behind the foil. However, occasionally an α particle was deflected at a large angle hitting the screen far from the beam's center line. In a few cases α particles were knocked backwards away from the foil in the direction from which they had come. This was completely unexpected. Rutherford later commented:

> "It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

An α particle had to hit a massive object head-on in order to be knocked backwards. What could such an object be? The object had to be small since most of the α particles missed the object passing through the foil without being deflected. The smaller the object was the harder it would be to hit. Since very few α particles were deflect, the particle had to be extremely small.

A gold atom is far more massive than an α particle, massive enough to cause an α particle to be knocked backwards in a head-on collision. To agree with the experimental results most of the atom's mass had to be concentrated in the small object. Rutherford visualized this object residing at the center of the atom. With most of the atom's mass residing in this object, which Rutherford called the nucleus, the remainder of the atom had to be mostly empty space. It was known from Thomson's experiments that the mass of an electron was nearly 2,000 times less than that of a hydrogen atom, and even smaller compared to the mass of a gold atom. From this knowledge Rutherford concluded that most of an atom's volume had to consist of electrons orbiting the massive, but very tiny, nucleus at the center of the atom much like the planets of the solar system orbit the Sun. The planets are held in their orbits by the Sun's gravitational force. The electrons had to be held in their orbits around the nucleus by the electrostatic force between the positively charged nucleus and the negative electrons.

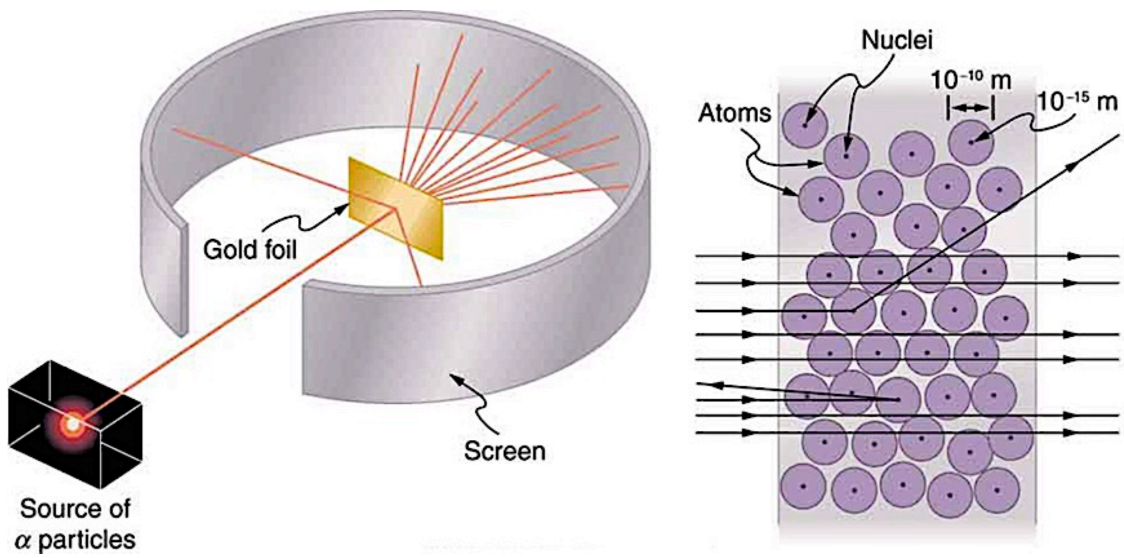


Figure 15 Rutherford's Gold Foil Experiment
(source: https://courses.lumenlearning.com/physics)

Rutherford's colleagues published the results of the experiment itself in 1909. However, like Thomson before him, Rutherford was reluctant to accept the radical ideas that the experiment seemed to suggest. It took him two more years to convince himself that the experiment was correct and to understand its meaning.

In 1911, Rutherford published his analysis together with a proposed model of the atom (Figure 16). Later the size of the nucleus was determined to be about 10^{-15} m, or 100,000 times smaller than an atom. This implied that the density of the nucleus was huge, on the order of 10^{15} g/cm^3, vastly larger than any macroscopic matter. Also implied by the experiment was the existence of a previously unknown force that could over come the repulsive Coulomb force to hold the positively charged protons together in the nucleus.

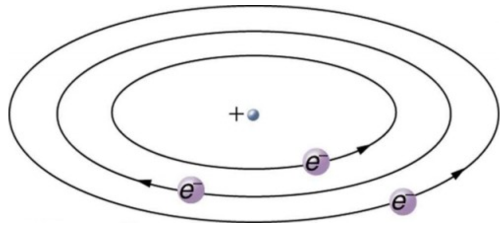


Figure 16 Rutherford's planetary model of an atom
(source: https://courses.lumenlearning.com/physics

2.6 Danish Physicist Niels Bohr

Rutherford's atomic model along with his supporting experimental evidence was one of his great scientific contributions. However, it received little attention beyond Manchester.

In 1913 Danish physicist Niels Bohr (1885 – 1962) showed the importance of Rutherford's work. Bohr visited Rutherford's laboratory the year before and became a faculty member at Manchester from 1914 to 1916.

Rutherford's atomic model had two serious defects in terms of classical physics.

First, the atom depicted in Rutherford's model was theoretically unstable. According to Maxwell's electromagnetic theory, the electrons in Rutherford's model should accelerate as they orbited the positively charged nucleus causing them to radiate electromagnetic energy. As they lost energy through radiation their orbits should continuously decrease as they spiraled into the nucleus causing the atom to collapse in about 10 picoseconds.

Second, the frequency of the radiated energy would be the same as the electron's orbital frequency. An electron's orbital frequency would continuously increasing as it spiraled into the nucleus causing the frequency of its radiated energy to also continuously increase.

These two conclusions completely disagreed with experimental result. First, atoms were stable. Second, from the line spectrums collected over many years, atoms did not radiate energy over a continuous spectrum but instead at discrete frequencies.

Figure 17 Danish physicist Niels Bohr (source: Pinterest)

In 1913 Bohr proposed a new atomic model, based on Rutherford's planetary scheme, that provided convincing arguments for the stability of atoms as well as their spectra. Like Rutherford, Bohr assumed that electrons orbited around the nucleus under the influence of electrostatic attraction from the nucleus. However, Bohr made several assumptions which violate classical physics but which were immensely successful in explaining many properties of the hydrogen atom:

- Instead of orbits at continuously increasing distance from the nucleus, only specific orbits at discrete distances from the nucleus were allowed.

- The only allowed orbits were those for which the orbital angular momentum L of an electron was an integer multiple of $h/2\pi$, that is

  $$L = n\frac{h}{2\pi} \qquad n = 1, 2, 3, \cdots$$

  where h is Planck's constant (h = $6.62607004 \times 10^{-34}$ m^2 kg / s) and n is called the principal quantum number.

- As long as an electron was in a permitted orbit, it would not radiate electromagnetic energy.

- Emission or absorption of radiated energy could take place only when an electron jumped from one allowed orbit to another. The radiation involved in an electron's transition from one orbit of energy E_n to another orbit with energy E_m was in the form of photons where the energy of a photon is

  $$hv = E_m - E_n$$

  v being the frequency of the radiated energy.

Consequently, a hydrogen atom absorbed energy only when an electron jumped to a higher orbit, the frequency of the absorbed light being:

$$v = \frac{E_m - E_n}{h}$$

Thus light shining through hydrogen gas would create a black line at a frequency v since the hydrogen gas absorbed light at that frequency. Similarly, an atom emitted radiation only when an electron jumped to a lower orbit creating the emission line spectrum shown in Figure 18.

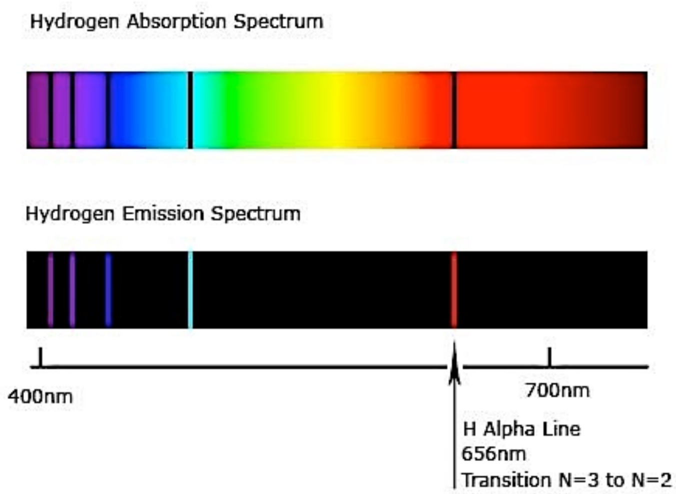


Figure 18
Hydrogen atom
Absorption and
Emission spectrum
(source: Quora)

The energy E_n for the n^th orbital of the hydrogen atom, known as Bohr energy, is

$$E_n = -\frac{e^2}{8\pi\varepsilon_0}\frac{1}{r_n} = -\frac{m_e}{2\left(\frac{h}{2\pi}\right)^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{1}{n^2} = -\frac{\mathcal{R}}{n^2}$$

where

n = the nth orbital (prime quantum number)

e = the charge on an electron

ε_0 = electric permittivity of free space = 8.854 x 10^{-12} farad per meter

r_n = the radius of the nth orbital from the nucleus

m_e = mass of an electron

$\mathcal{R}$ = Rydberg constant

$$\mathcal{R} = \frac{m_e}{2\left(\frac{h}{2\pi}\right)^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 = 13.6\ eV$$

Since $E_n = -\mathcal{R}/n^2$ the energy of each orbital of a hydrogen atom is determined by the value of the quantum number n. The negative sign for E_n indicates that the electron is bound to the atom.

The structure of the hydrogen atom energy spectrum E_n is shown in Figures 19 and 20.

n = 1 is defined as the ground state, the lowest energy level (most negative) that an orbital can obtain. As n increases the energy level separations decrease rapidly as can be seen in Figure 19. Since n can have any integer value from $n = 1\ to\ n = \infty$, the energy spectrum of the hydrogen atom has an infinite number of discrete energy levels. In the ground state (n = 1) the atom has an energy of

$$E_1 = -\mathcal{R} = -13.6\ eV \qquad and\ a\ radius\ \ a_0$$

The states n = 2, 3, 4, …….. are referred to as excited states since their energies are greater (less negative) than the ground energy state.

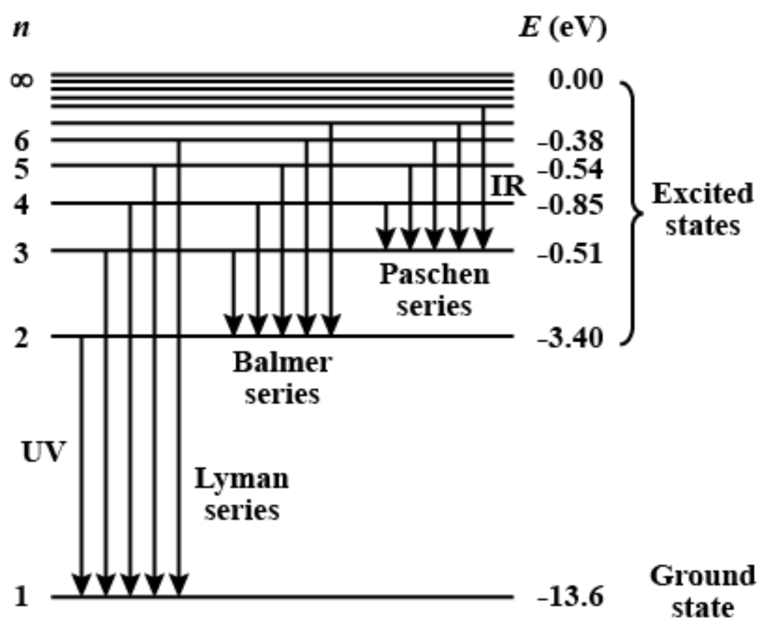


Figure 19
Energy levels for hydrogen atom
(source: Topper)

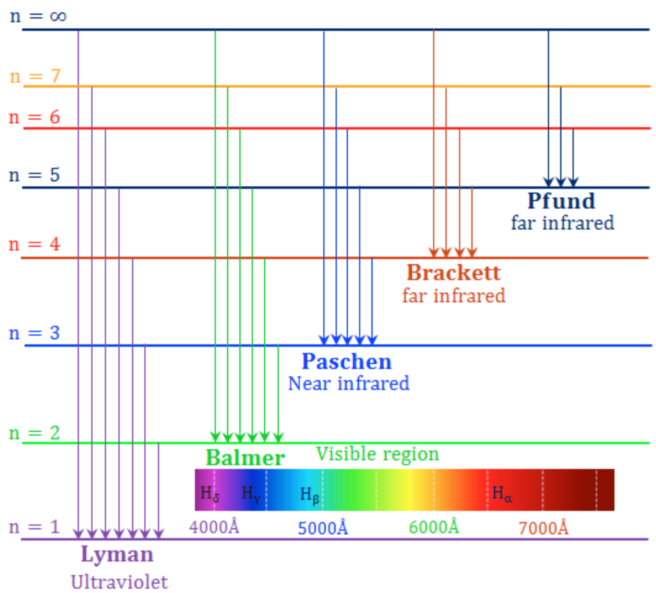


Figure 20
Hydrogen atom
Spectral series
(source: Pinterest)

Notice in Figures 19 and 20 that electrons are not restricted to jumping only between neighboring energy levels. They can jump between any two energy levels. In Figures 19 and 20 the Lyman spectral lines are formed by electrons dropping to the ground state (n = 1) from higher energy levels. The Lyman lines are in the ultraviolet part of the spectrum. The Balmer spectral lines are formed by electrons dropping to the n = 2 energy level. These lines are in the visible part of the spectrum. The Paschen lines (electrons dropping to n = 3) are also in the visible region where as the Brackett and Pfund lines are in the infrared region.

When the quantum number n is very large $(n \to +\infty)$ the atom's radius r_n will also be very large. However, its energy values go to zero $(E_n \to 0)$ as illustrated in Figure 19. This means that the proton and the electron are infinitely far apart and hence they are no longer bound together. That is, the atom is ionized with the electron becoming a free particle capable of having any amount of kinetic energy.

In 1922 Bohr received the Nobel Prize in Physics for his foundational contribution to the understanding of atomic structure and quantum theory.

2.7 Henry Moseley

In 1913 English physicist Henry Moseley (1887 – 1915) used an early form of X-ray spectrometry to measure the X-ray spectra produced by various chemical elements when they were bombarded by electrons inside an evacuated electron tube.

Figure 21 Henry Moseley

(source: Museum of the History of Science, University of Oxford)

Moseley discovered that the square root of the X-ray frequency emitted by the target element was proportional to Z–1, where Z was a whole number representing the charge on the nuclei of the target element atoms. This became known as Moseley's Law. In tern, the symbol Z became known as the atomic number for an element.

Before Moseley and his law, chemical periodic tables were arranged primarily by atomic weight. While atomic numbers were used to identify elements in the table, they were thought of as being semi-arbitrary vaguely increasing with atomic weight but not strictly defined by it. Moseley's discovery showed that atomic numbers were not arbitrary but instead dictated by each atom's positive nuclear charge. Moseley proposed that each successive element in the periodic table had a nuclear charge exactly one unit greater than its predecessor. Moseley's experiment provided a way to determine what the nuclear charge for each element actually was.

Moseley's law caused the periodic table to be revised. In some cases the position of adjacent elements in the table were reversed. In other cases holes were found in the table indicating that there were elements not yet discovered. Based on atomic weights chemists suspected that there were missing elements because of large jumps in the weights of adjacent elements that occurred at various places throughout the table. Moseley's law not only confirmed their suspicions but also indicated how many elements were missing in each hole.

During World War I Moseley left his research work at the University of Oxford and join the British army as a telecommunications officer. On 10 August 1915 Moseley was killed during the Battle of Gallipoli in Turkey.

2.8 Discovery of the Proton

In 1917 Rutherford found that small amounts of hydrogen were produced when nitrogen gas was bombarded with alpha particles. He did not report the experiments until 1919, 1920, and later in 1925. Rutherford initially assumed an alpha particle merely knocked a positive charge out of the nitrogen nucleus, turning it into carbon according to the equation

$$_7N^{14} + \alpha \rightarrow {}_6C^{14} + \alpha + H^+$$

However, after observing Blackett's cloud chamber images in 1925, Rutherford realized that the alpha particle was absorbed by a nitrogen atom. After capturing the alpha particle, the nitrogen atom ejected a hydrogen nucleus transforming the nitrogen into heavy oxygen, not carbon. In equation form

$$_7N^{14} + \alpha \rightarrow {}_8O^{17} + H^+$$

This was the first reported nuclear reaction. Note that the above equations are more exact than Rutherford understood at the time since neutrons had not yet been discovered.

Rutherford knew that hydrogen was the simplest and lightest element containing only a single positive charge. Influenced by earlier theories suggested by others, Rutherford suspected that hydrogen was the building block of all other elements. Discovering that the hydrogen nucleus was present in the nitrogen atom convinced Rutherford that the hydrogen nucleus H^+ was an elementary particle present in all atoms.

Rutherford described his nitrogen experiment at the 24 August 1920 meeting of the British Association for the Advancement of Science. As part of his presentation he explained his reasons for concluding that the hydrogen nucleus H^+ was an elementary particle. During the meeting Oliver Lodge asked Rutherford to recommend a name for the hydrogen H^+ nucleus that would distinguish it from a neutral hydrogen atom. Several names were suggested before arriving at the name "proton" from the Greek word meaning first. The name was accepted at the meeting. Thus protons make up the nuclei of all atoms, the nucleus of a neutral hydrogen atom consists of a single proton.

2.9 Discovery of the Neutron

By the mid 1920s it was known that the atom consisted mostly of empty space with a tiny nucleus composed of positively charged protons located at its center. Most of the atom's volume was occupied by widely scattered negatively charged electrons in various orbits around the nucleus giving the atom a neutral charge. For instance, helium was known to have an atomic number of 2 (its nucleus contained 2 protons) but an atomic weight of 4. Where did the additional mass come from? Some scientists speculated that the nucleus actually contained additional protons with an equal number of electrons within the nucleus to cancel out the positive charge of these additional protons. In 1920 Rutherford proposed that a proton and an electron could combine in the nucleus to form a new neutral particle. However, there was no direct evidence of this.

In the early 1930s James Chadwick (1891 – 1974) discovered the neutron.

Chadwick studied physics at the University of Manchester where he worked with Rutherford on various radioactivity studies. In 1914 he went to Germany to study with Hans Geiger (the inventor of the Geiger counter). However, World War I soon broke out. Chadwick was captured by the Germans and spent the next four years in a prisoner of war camp. While there Chadwick and some fellow prisoners established a science club and convinced the guards to allow them to set up a small laboratory. After the war, Chadwick returned to Cambridge University where he completed his PhD in 1921. In 1923 he was appointed assistant director of the Cavendish Laboratory.

Around 1930 a number of researchers began studying the highly penetrating radiation emitted by beryllium when it was bombarded with alpha particles. Some scientists believed that the radiation was due to high energy photons being emitted by the beryllium.

Figure 22 James Chadwick (source: AIP Emilio Segre Visual Archives)

Researchers Frédéric and Irène Joliot-Curie found that the radiation from beryllium knocked loose protons from a paraffin wax target. Joliot-Curie, believed the beryllium radiation must be high energy gamma ray photons. But Chadwick didn't think this explanation was correct. He reasoned that photons having zero mass couldn't knock loose particles as heavy as protons. Chadwick tried similar experiments himself in 1932. From these experiments he became convinced that the beryllium radiation was composed of neutral particles about the same mass as a proton. In addition to paraffin wax, he also experimented with helium, lithium, and nitrogen targets. These additional experiments helped him determine that the new particle's mass was just slightly more than the mass of a proton.

Chadwick concluded that the neutral charge of this new particle would account for its high penetration ability allow it to penetrate much further into a target than protons.

In February 1932 Chadwick published a paper titled "The Possible Existence of a Neutron." In this paper he proposed that the highly penetrating beryllium radiation was more likely to consist of neutral particles rather than gamma ray photons. A few months later, in May 1932, Chadwick submitted the more definite paper titled "The Existence of a Neutron." By 1934 it was widely accepted that, along with electrons and protons, the newly discovered neutron was in fact the third fundamental atomic particle.

Neutrons accounted for the missing mass in atoms. The nucleus of the helium atom, with an atomic number of 2 and a mass of 4, was now know to consist of 2 protons plus 2 neutrons. That is, 2 protons (helium's atomic number) + 2 neutrons equals a mass of 4 atomic units. Furthermore, the composition of alpha particles was finally known. Alpha particles also consisted of 2 protons and 2 neutrons since it was known for quite some time that an alpha particle was simply the nucleus of a helium atom.

2.10 Atom Defined by Quantum Mechanics

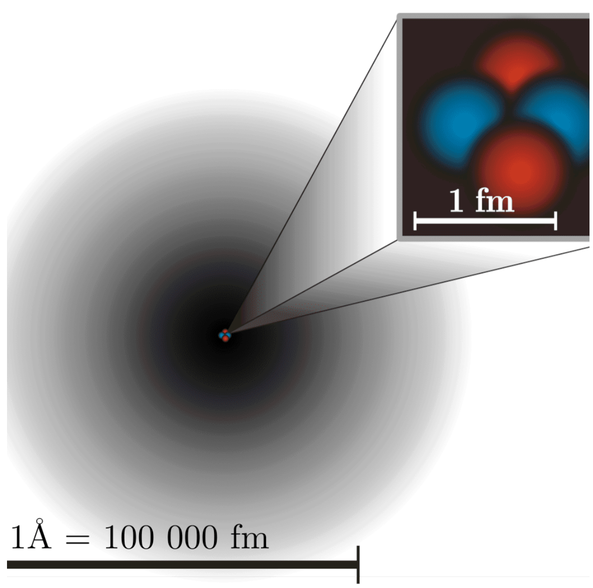
Bohr's model was quite an exciting success at the time. However, it quickly became clear that there were many inconsistencies associated with the model. The model did a good job describing the characteristics of the hydrogen atom and predicting the occurrence of hydrogen spectral lines. However, it could not accurately predict the characteristics of heavier atoms or the complex spectral lines associated with them. It became apparent that more complex models of the atom were needed. However, it was also clear that Bohr's model with its quantized energy levels and permitted electron shells was leading physicists in the right direction. In the years that followed physicist including Arnold Sommerfield, Otto Stern and Walther Gerlach, Wolfgang Pauli, Werner Heisenberg, and others produced increasing more complex and accurate models of the atom driven by the quickly developing field of quantum mechanics. Today's model of the atom, which arises from quantum mechanics, is completely different from Bohr's simplistic model, but Bohr pointed the way.

3 The Structure of an Atom

Today we know that an atom consists of a small dense central nucleus, composed of positively charged protons and electrically neutral neutrons, surrounded by a probabilistic cloud of negatively charged electrons (Figure 23). The size of typical atoms (the diameter of the atom's electron cloud) range from 62 pico-meters (pm) for a helium atom to 520 pm for a cesium atom, a pico-meter being $1 \cdot 10^{-12}$ meters. As a reference, one angstrom is a length of 100 pm. The diameter of the electron cloud is typically 100,000 times larger than the nucleus. So relatively speaking, the nucleus is a small "dot" at the center of a huge spherical electron cloud.

3.1 Atomic Number and Mass

The Atomic Number of an element is equal to the number of protons in the element’s nucleus. The number of protons defines the type of atom. For example, gold with an atomic number of 79 has 79 protons in its nucleus. Lead, atomic number 82, has 82 protons. A difference of 3 protons makes quite a difference. The atomic number determines the element’s position in the periodic table. In the periodic table shown in Figure 24, the atomic number of each element is in the upper left corner of the box representing that element. All atoms of a particular element must have the same number of protons. The diameter of a photon is about 0.842 femtometres = 0.842 (10^{-15}) m.



1 fm = 1 femtometer = 10^{-15} m

Figure 23 Structure of an Atom (source: thoughtco.com)

The mass of a neutron is slightly heavier than a proton. In contrast, the mass of an electron is only $\frac{1}{1{,}836}$ that of a proton. Consequently, 99.97% of an atom's mass is concentrated in its nucleus. The mass of an atom can be calculated knowing that:

The mass of a proton is $1.6726231 \cdot 10^{-27}$ kg ,
The mass of a neutron is $1.6749286 \cdot 10^{-27}$ kg (slightly heavier than a proton), and
The mass of an electron is $9.1093897 \cdot 10^{-31}$ kg.

The Atomic Mass of an atom is defined as the total number of protons and neutrons in its nucleus. The mass of all of an atom's electrons added together is negligible.

The Atomic Mass of an atom, as given in the Periodic Table, generally is not an integer number. For example, the atomic mass listed for the carbon atom (atomic number 6) is 12.01 while that for aluminum (atomic number 13) is 26.98. The reason for non-integer atomic mass is that the number of neutrons in the nucleus can vary within certain limits. The atomic mass listed for an atom in the periodic table is its average mass.

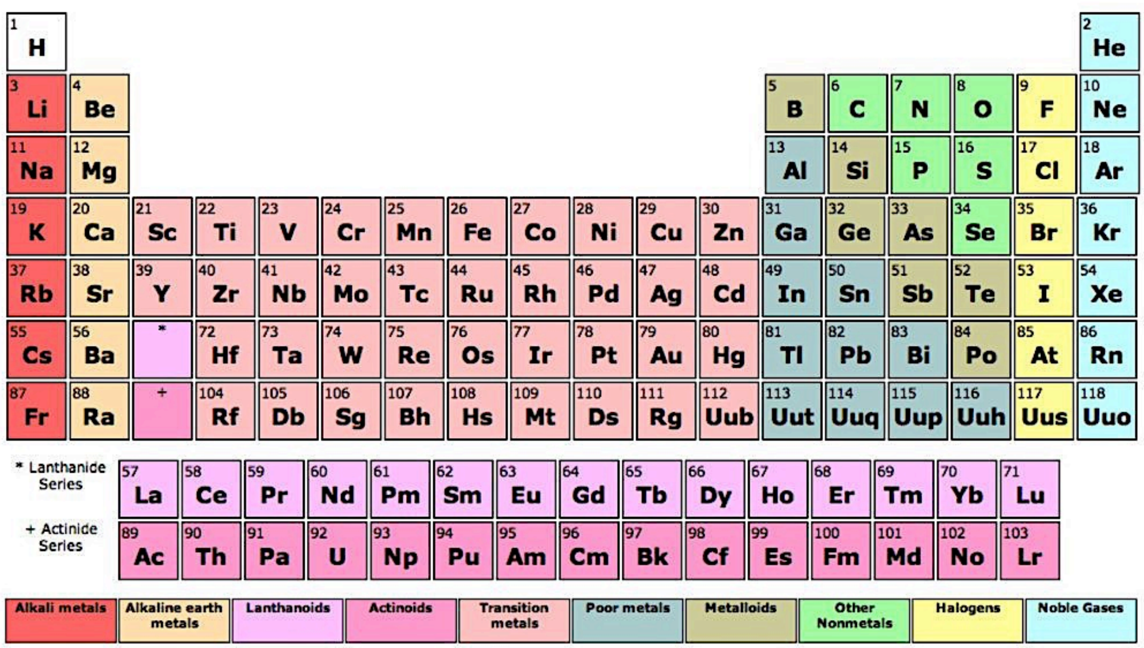


Figure 24 Periodic Table of Elements

The standard chemical nomenclature used in specifying atoms is ${}_{a}X^{b}$ where X is an atom, "a" is the atomic number of the atom (the number of protons in its nucleus), and "b" is its mass number (the number of protons plus neutrons in its nucleus).

3.2 Isotopes

The most common atoms have relatively light nuclei. They also tend to have the same number of protons and neutrons. A typical carbon atom, $_6C^{12}$ for example, has 6 protons and 6 neutrons with an atomic mass of 12. However Carbon-14, $_6C^{14}$, has 6 protons and 8 neutrons. Its atomic mass is 14. An atom that has more or fewer neutrons than the typical number is called an isotope. Carbon-14 is an isotope of the commonly occurring Carbon-12.

Atoms with heavy nuclei generally have more neutrons than protons, often considerably more neutrons. For example, uranium-238 $_{92}U^{238}$ has 92 protons and 146 neutrons.

3.3 Ions

In its electrically neutral state an atom has the same number of electrons and protons. That is, the total positive charge of the protons is exactly cancelled by the total negative charge of the electrons, leaving the atom with a net charge of zero.

An atom becomes an ion if this balance is upset. An atom that gains one electron becomes a negatively charged ion because it has one more electron than the number of protons. An atom may gain an electron by stealing it from some other atom or by capturing a free electron that is not attached to any atom. Similarly, a positive ion is formed if an atom looses an electron. In this case the electron may be lost as the result of high energy radiation bombarding the atom or an other atom stealing the electron.

An electron that breaks free from its parent atom becomes a independent entity, relegating its parent to a positive ion. No longer bound to any atom, a free electron travels randomly at high speeds through the voids between atoms, frequently colliding with them. An electron often rebounds from a collision darting off in some other direction. However, a collision can also result in an electron being captured by the atom or ion with which it collided.

The outer electrons in metals (particularly gold, silver, copper, and aluminum) are so loosely held that they easily break free from their parent atoms forming a vast sea of unattached electrons that give metals their characteristic high electrical conductivity.

Atoms are so far apart in Earth's upper atmosphere that free electrons can exist for quite some time before being captured by a positive ion or another atom. These upper atmosphere free electrons form the ionosphere critical to long range high frequency radio communications. A free electron is tiny compared to atoms and ions which are tens of thousands of times more massive.

3.4 Chemical Properties

Electrons determine the chemical properties of an atom. It is the bonding, or sharing of electrons between atoms that forms a molecule. A molecule is composed of various types of atoms bound to each other by chemical bonds. For example a water molecule H_2O consists of 2 hydrogen atoms bound to a single oxygen atom. Electrostatic force is the force holding together a molecule. Molecules are the smallest particles of a substance which retains the characteristics of that substance.

3.5 Creation of Atoms

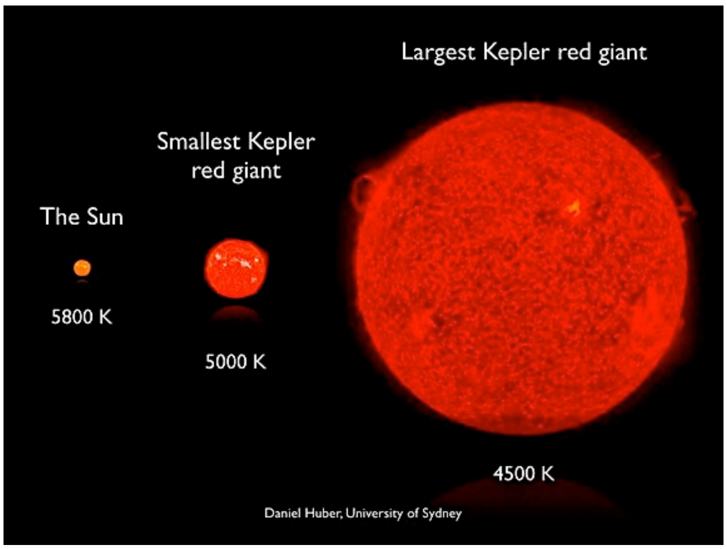
With the exception of hydrogen and some initial helium, all of the various elements are produce by thermonuclear fusion within the trillions upon trillions of stars that populate the universe, our Sun being one of them.

Stars are composed almost entirely of hydrogen. Our Sun consists of 71% hydrogen, 27% helium and 2% all other elements by mass. The immense size of a star produces an enormous gravitational field that holds planets in orbits around the star. The gravitational force within the star is so intense that two-thirds of its mass is squeezed into its core, the innermost 3% of its volume, creating enormous temperature and pressure. The temperature in the core is so high that all of the electrons are stripped from hydrogen and helium atoms leaving only hydrogen and helium nuclei (for hydrogen a single proton and 2 protons + 2 neutrons for a helium nuclei). The core is extremely dense with hydrogen and helium nuclei compressed tightly together. In contrast, the density of hydrogen gas in the outer layers of a star, the part that we can actually see, is often much less than air pressure we experience on Earth.

The immense temperature and pressure within the core ignites a thermonuclear reaction (actually a chain of 3 different reactions) that fusses hydrogen nuclei together to form helium. The energy produced by this series of reactions balances the star's gravitational force maintaining the star in a stable condition for billions of years (keeping the star at a constant size and internal temperature and pressure).

Gradually over time the hydrogen in the core is depleted leaving only helium. The remainder of the star, outside the core, is still composed almost entirely of hydrogen. But radiation from the thermonuclear reaction prevents this hydrogen from replenishing the supply of hydrogen in the core. Starved of hydrogen, thermonuclear fusion diminishes. The core begins to collapse under the weight of the star's outer layers. Pressure and temperature increase within the shrinking core until at about a 100 million degrees thermonuclear fusion of helium into carbon, nitrogen, and oxygen begins. The fusion of helium produces only 5% of the energy generated by hydrogen fusion. The helium fuel also runs out much quicker, typically in 1 to 2 million years. Over this period of time, the heat produced by helium fusion causes the star to balloon into a bloated red giant 100 times its original size (Figure 25). When our Sun becomes a red giant billions of years from now, it will engulf Mercury and Venus and turn Earth into molten rock. A red giant is so bloated that its average density is often only one thousandths that of the air we breath.

The star collapses when the helium in its core is exhausted. The outer portions of the bloated star are blown off into a planetary nebula surrounding the collapsed star (Figure 26). The star becomes a tinny blue-white dwarf, about the size of the Earth, with a density measured in tons per square inch. A white dwarf may last for tens of billions of years. Eventually the dwarf will also die leaving an extremely dense “black rock”.



(source: Daniel Huber University of Sydney)

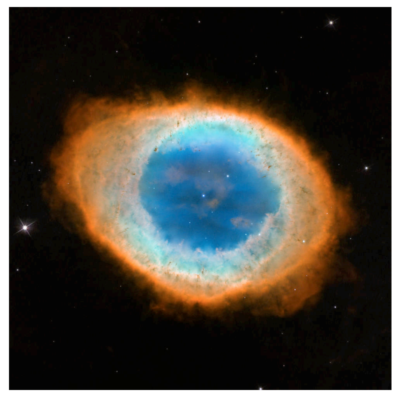


Figure 25 Red Giant Star vs the Sun

Figure 26 Planetary Nebula

The thermonuclear fusion process in a star several times more massive than the Sun continues beyond the fusion of helium into carbon. When the helium fuel runs out, the core shrinks under the star’s tremendous weight and heats up further. At around 600 million degrees carbon thermonuclear reaction ignites fusing carbon into oxygen, neon, and magnesium. In very massive stars the sequence of thermonuclear fusion continues up through the table of elements until the core becomes iron, cobalt, and nickel. At this point core temperature has reached several billion degrees, the core size has shrunk to around 1,000 Km, and its density is about a billion kilograms per cubic meter. The core becomes unstable, collapses, and the entire star implodes into a super nova with enormous brilliance.

However a star dies, the hydrogen, helium, and other elements present in its outer layers are blown off becoming a gas nebula from which new stars and planets form.

3.6 Chemical Composition of the Universe

Most of the universe is composed of hydrogen. In fact, when considering all of the atoms in the universe, 92% of them are hydrogen. Hydrogen is the smallest atom with one proton in its nucleus and a single electron. Yet, hydrogen accounts for 74% of the atomic mass in the universe. 7.5% of all atoms are helium. Helium, with 2 protons and 2 neutrons, accounts for 24% of the mass. Together hydrogen and helium make up 99.8% of the atoms and 98% of the atomic mass in the universe. All of the other elements account for the remaining number of atoms and mass. The following table summarizes the composition of the universe.

Element	**Atoms**	**Atomic Mass**
Hydrogen	92%	74%
Helium	7.5%	24%
Oxygen		1%
Carbon		0.5%
All other elements		0.5%

Composition Of The Universe

It is interesting that the most abundant elements in the universe, hydrogen, oxygen, and carbon are also the critical elements required for life. The universe is structured for the creating life.

The composition of the Earth is much different. Only about 0.2% of the mass in the Earth's interior is hydrogen and it accounts for only about 10% of the ocean mass. Helium is almost non-existent on Earth. Helium was first discovered on the Sun and later trace amounts found on Earth.

4 Elementary Particles

An electron is an elementary particle. That is, it has no internal structure. It is not composed of any smaller particles. An electron is an electron and it has an electrical charge of -1.

Protons and neutrons are not elementary particles.

A proton is composed of two Up Quarks, each with a +2/3 charge, and one Down Quark with a charge of -1/3. The net charge of a proton is thus:

1 proton = 2 Up Quarks + 1 Down Quark = $2(2/3) - 1/3 = 4/3 - 1/3 = 3/3 = +1$.

A neutron consists of 1 Up Quark (+2/3 charge) and two Down Quarks (each with a -1/3 charge). The net charge of a neutron is thus 0.

Up and down quarks are elementary particles. Like electrons, they have no internal structure.

5 Forces of Nature

In all of nature there are only 4 types of forces:

- The strong force,
- Electromagnetic force,
- The weak force, and
- Gravitational force.

The relative strengths of the four forces is shown in the following table in which the weak force has been arbitrarily assigned a value of 1.

Force	Relative Strength
Strong	10,000
Electromagnetic	100
Weak	1
Gravity	$7 \cdot 10^{-34}$

The strong force is the strongest force in nature. It holds together quarks within protons and neutrons and holds protons and neutrons together in the nucleus of an atom. The strong force operates over an extremely short distance which is about the diameter of a proton. Two particles must be almost touching to be affected by the strong force. However, when two particles get close enough, the strong force "kicks in" dominating all other forces in nature. Since the strong force is one hundred times stronger than the electromagnetic force, it is able to hold together protons within a nucleus even though the positively charged protons, under the influence of the electromagnetic force, try to repel one an other.

The electromagnetic force is the next strongest force. The electromagnetic force holds together atoms binding the cloud of negatively charged electrons to the positively charged nucleus. The electromagnetic force also binds together atoms into molecules and molecules into all of the more complex structures that we are accustom to including trees, oceans, and our human body. The electromagnetic force is the primary force underlying most of engineering, physics, chemistry, and biology. The electromagnetic force has an infinite range. The electromagnet force between two electrically charged objects is inversely proportional to the square of the distance separating them, but never goes to zero. Two electrons on opposite sides of a galaxy still repel each other even though the force between them is infinitesimally small. The electromagnetic force causes most large objects in the universe to be electrically neutral. If an object is electrically positive, it will over time attract and capture a sufficient number of electrons to become electrically neutral. A negatively charged object will eventually eject enough electrons to also become electrically neutral.

The weak force has the shortest range of all the forces. Its range of influence is only about 1% the diameter of a proton. It is extremely unlikely that two particles will get close enough to interact via the weak force. Thus reactions that occur as a result of the weak force occur at an extremely slow rate. However, the weak force is responsible for radioactive decay and, under certain conditions, the transformation of a proton into a neutron and the conversion of a neutron into a proton. This conversion process between protons and neutrons enables the production of all elements other than hydrogen. Radioactive decay provides much of the heat required to keep the Earth's iron core molten. A molten iron core is extremely important since it is responsible for Earth's significant magnetic field that largely shields Earth's atmosphere from erosion by the solar winds emanating from the Sun. In contrast, Mars has a solid core and little or no magnetic field. The absence of a protective magnetic field caused the atmosphere of Mars to be blown away by the solar winds long ago.

Gravity is incredibly weak in comparison to the other three forces, and yet it is the most important. In relative terms, the strength of gravity is only $7 \cdot 10^{-34}$ compared to the weak force, the next smallest force. However, the range of gravity is infinite and it attracts **<u>all</u>** types of mater. Thus gravity accumulates! The gravity associated with a small particle attracts other particles and they attract it, forming a larger body. The gravitational force associated with a large body is the sum of the gravitational force of each of its component particles. As a body grows in size, so does its gravitational force pulling in more and more material, leading eventually to the development of planets, stars, and galaxies. Gravity holds together galaxies, galactic clusters, and solar systems. It makes stars and planets rounds. It holds us, the oceans, and most other things that we are familiar with in our daily lives to the surface of the Earth. It causes weaken buildings to collapse, mountains to erode, and ultimately determines the fate of the universe. Thus, despite its intrinsic weakness, the infinite range of gravity and its attraction force exerted on all forms of mater cause gravity to dominate all other forces on a large scale.

6 The Electron Cloud

Returning to the structure of an atom, it is tempting to think of an atom as somewhat resembling a solar system with the nucleus being analogous to the Sun and the electrons representing the planets orbiting around the Sun (the Rutherford and Bohr models). However, this analogy is not correct. While the nucleus is approximately at the center of an atom, the location of a particular electron can only be determined statistically. That is, there is a certain probability that an electron exist somewhere within a shell surrounding the nucleus.

All of the electrons in the electron cloud are bound to the atom and occupy discrete electromagnetic energy levels, or orbitals, within the atom. These energy levels are described in terms of quantum mechanics. A particular energy level (orbital) can only be occupied by a single electron. This discovery is know as the Pauli exclusion principal. Hense, an electron can only occupy a specific orbital. An electron can not have an energy level that falls between that of two adjacent orbitals. An electron can, however, transition from one orbital to another, provided that an other electron does not already occupy that orbital. An electron transitions from one orbital to another by absorbing or emitting a photon who's energy exactly matches the energy difference between the two orbitals.

6.1 Spectral Lines

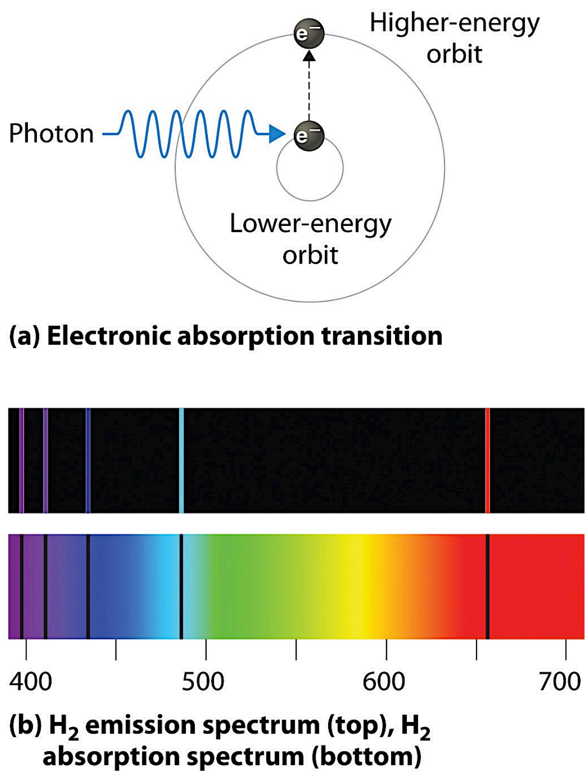
The energy E of a photon is equal to its frequency f times Planck's constant h, that is:

$$E = h \cdot f$$

The difference in energy between orbitals thus appears as distinct bands in the electromagnetic spectrum. The emission and absorption spectrum of a hydrogen atom is shown below (Figure 27).

Shining white light, or light with even a broader energy specturm, through hydrogen gas ensures that within that spectrum there are photons at exactly the correct energy level (frequency) to excite a hydrogen atom electron causing it to jump to a higher energy orbital. Dark absorption lines appear in the continuous spectrum of light after it has passed through the hydrogen gas. These dark lines occur because the photons at these frequencies have been absorbed by the hydrogen gas atoms. The presence of multiple absorption lines indicates that the electrons of the hydrogen atoms did not all jump to the same energy level. Some jumped to higher energy levels than others.

Atoms are most stable when their electrons are at their lowest possible energy levels, know as the ground state. Consequently, once excited, the hydrogen gas electrons spontaneously drop back to their low energy level. In the process they emit photons in random directions at the same frequencies as the photons they previously absorbed. These photons create an emission spectrum which can be seen when the hydrogen gas is viewed from the proper direction, that is a direction in which source of white light is not in the background.



Figue 27 Hydrogen emmission and absorption spectrum (source: saylordotorg.github.io)

Shining broad band light through other atoms heated to their gaseous state creates similar absorption and emission spectrum. Each atom has its own unique spectrum which serves as "a finger print" identifying the type of atom. The continuous spectrum of energy from the Sun, and other stars, contain absorption lines that reveal the type of atoms present in the outer regions of the Sun, as illustrated below. However, some of these absorption lines are the result of photons being absorbed by atoms in the Earth's atmosphere. For this reason, observing the spectrum of stars is best performed from spacecraft.

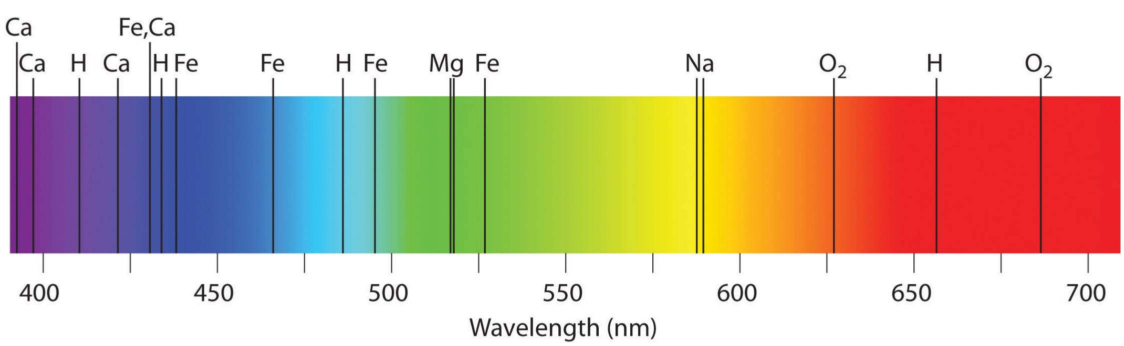


Figure 28 Absorption spectrum of the sun (source: Continuous Spectrum)

6.2 Electron Orbitals

Each orbital in an atom is uniquely identified by three quantum numbers n, l, and m. These numbers corresponds to the energy, angular momentum, and magnetic moment of the electron occupying an orbital. A maximum of two electrons can occupy an orbital each with its own spin quantum number.

The principal quantum number n identifies the energy of the electron and determines the mean distance of the electron from the nucleus. All electrons with the same value of n are the same average distance from the nucleus. For this reason each value of n is said to be an electron shell with the ability to contain one or more electrons. The quantum number n is always a positive integer. In theory, n can be any positive integer. However, for naturally occurring atoms $1 \leq n \leq 7$. The energy level of an electron increases as the value of n increases. The electron shell n = 1 is the lowest energy level shell and the shell closest to the nucleus. Shell n = 2 is the next higher energy level shell. Thus, electrons in shell n = 2 have a higher energy level than electrons in shell n = 1.

The azimuthal quantum number l describes the angular momentum of an electron. This quantum number is a non-negative integer. Within a specific electron shell n, the values for l range between $0 \leq l \leq n-1$. For example, if n =3 then l can have values of $l = 0,\ l = 1,\ and\ \ l = 2$. Each value of l is a subshell of n.

The magnetic quantum number m describes the magnetic moment of an electron. This quantum number is always an integer. Within a specific subshell l, the values for m range between $-1 \leq m \leq +1$. The table below shows the allowed values of m for $1 \leq n \leq 5$. Each cell in the chart corresponds to a specific subshell and indicates the magnetic quantum numbers m available for that subshell. As an example, for subshell n = 3 and $l = 1$ the magnetic quantum numbers -1, 0, and +1 are available. Cells which are blank in the chart indicate that subshell does not exist.

	$l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$
n = 1	m = 0				
n = 2	m = 0	-1, 0, 1			
n = 3	m = 0	-1, 0, 1	-2, -1, 0, 1, 2		
n = 4	m = 0	-1, 0, 1	-2, -1, 0, 1, 2	-3, -2, -1, 0, 1, 2, 3	
n = 5	m = 0	-1, 0, 1	-2, -1, 0, 1, 2	-3, -2, -1, 0, 1, 2, 3	-4, -3, -2, -1, 0, 1, 2, 3, 4

Each electron has a spin quantum number s describing the electron spin. The spin quantum number s can have a value of either $+\frac{1}{2}$ or $-\frac{1}{2}$ corresponding to "spin up" or "spin down".

In chemistry, the electron shells and subshells are given names according to the following equation.

$$n(type)^y$$

In this equation n is the principal quantum number n designating the energy level of the shell. The term "type" is a lower case letter denoting a subshell of the orbital. The subshells arranged from lowest to highest energy level are s, p, d, f, and g. These subshells correspond to values of the angular moment quantum number l . Specifically $s \rightarrow l = 0,\ \ p \rightarrow l = 1,\ \ d \rightarrow l = 2,\ f \rightarrow l = 3\ \ and\ \ g \rightarrow l = 4$. The letter y denotes the number of electrons present in the orbital. For example, the orbital $1s^2$ is the lowest energy level orbital (n = 1), it angular momentum number $l = 0$, and in this example it is occupied with two electrons, the maximum allowed for this subshell. The orbital $2p^4$ is at the second energy level (n = 2), its angular momentum number $p \rightarrow l = 1$, and it contains 4 electrons, a maximum of 6 electrons are allowed.

When unperturbed, an atom exist in its lowest energy level state. That means the lowest energy subshells are filled with electrons first. An energy level diagram of the subshells up through 5s is shown below (Figure 29). In this diagram, electrons fill successive subshells beginning at the bottom of the diagram (with the 1s subshell) and working up. Notice that subshell 4s is a lower energy subshell than 3d. Consequently, electrons fill the 4s subshell before beginning to fill the 3d subshell. Subshell 5s is also a lower energy subshell than 4d. In addition, notice that the energy level of a free unbound electron (an electron that has broken free from the atom's electron cloud) is much higher than the energy level of any of the electrons bound to the atom.

The energy level of an electron is usually measured in electronvolts (eV). A bound electron that is at its lowest (most stable) energy level is said to be at its ground state. An electron is excited when it transitions to a higher energy level. An electron’s energy level increases as n increases because its average distance from the nucleus increases. The energy associated with an electron's l quantum number is not caused by the electrostatic potential of the electron's position relative to the nucleus, but instead by the interaction of the electron with other electrons.

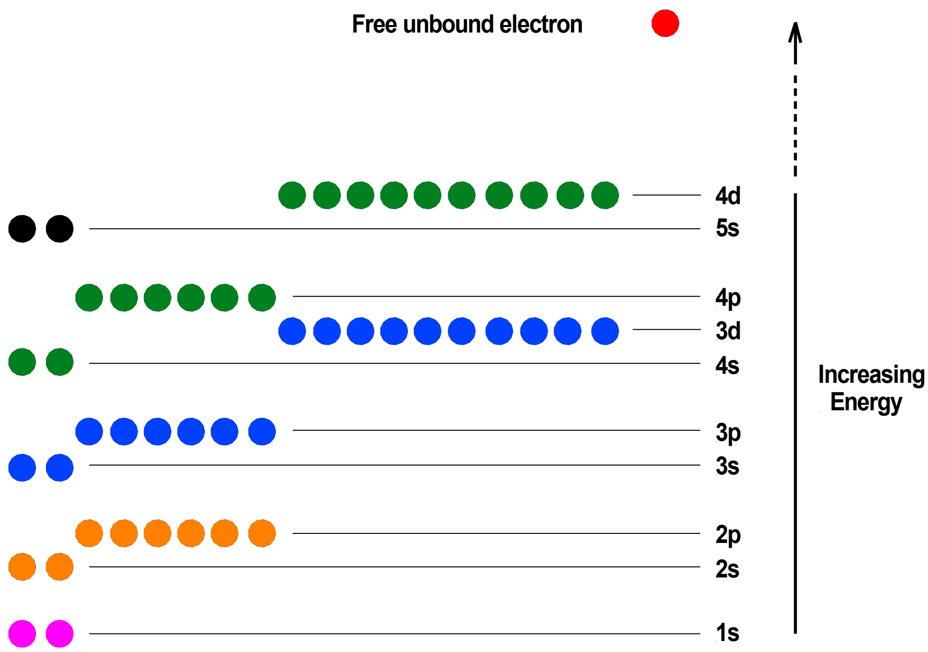


Figure 29 Energy Levels – showing the number of electrons which may fill each subshell

7 Chemical Bonds

A molecule is the smallest possible unit of a substance that still retrains the properties of that substance. A molecule consists of a unique number of specific types of atoms that are bound together in a particular way by either ionic or covalent bonds.

The outermost electron shell of an atom is known as the valence shell. It is the shell furthest from the nucleus and the highest energy level shell. The electrons that occupy this shell are called valence electrons.

All of the electron positions in the lower energy shells, the shells closer to the nucleus, must be completely filled before electrons can begin populating the valence shell. Consequently, the valence shell is typically only partially filled. An atom is at its lowest energy level when its valence shell is completely filled or empty.

An alkaline metal atom such as hydrogen, lithium, sodium, potassium, etc. (an atom on the left side of the periodic table) has a single valence electron. The electrostatic force that the positively charged nucleus exerts on this electron is very low due to the electron's position in the outer part of the electron cloud. However, the electrostatic force is sufficient enough to prevent the electron from simply drifting way from the atom. If the atom did lose this electron, its valence shell would be empty lowering the over all energy level of the atom. Consequently, this electron is very loosely held by the atom.

A transitional metal atom, an atom in the middle of the periodic table such as iron, cobalt, nickel, copper, and zinc has a small number of valence electrons. These electrons are also loosely held.

In contrast, an atom on the right side of the periodic table, i.e. a non-metal atom such as carbon, nitrogen, oxygen, phosphorus, sulfur, and chlorine has a valence shell which is nearly filled. Filling its valence shell by capturing stray electrons lowers the atom's energy level. Thus these atoms have a strong affinity for electrons.

Atoms on the far right of the periodic table such as helium, neon, argon, krypton, etc. have completely filled valence shells and are thus already at their lowest possible energy levels. These are the inert gases and do not react readily with other elements.

The number of valence electrons determines the chemical bonding characteristics of an atom.

7.1 Ionic Bond

An ionic bond is formed when an alkaline or transitional metal atom comes in contact with a non-metal atom. Because of its strong affinity for electrons, the non-metal atom "steals" a loosely held valence electron from the metal atom. The electron transfer creates two ions, a positively charged metal ion and a negatively charged non-metal ion. The electrostatic force between the two ions holds them in a strong ionic bond. The energy states of the two bound atoms is lower than either atom alone. Thus ionic bonds form quickly lowering the total energy of the resulting molecules.

When sodium and chloride react to form sodium chloride, $NaCl$, electrons are transferred from the sodium atoms (Na) to the chloride atoms (Cl) to form Na^+ and Cl^- ions as shown in Figure 30 below. The electrostatic force between the closely packed oppositely charged ions in solid sodium chloride results in very strong ionic bonding that has considerable thermal stability. Consequently, solid sodium chloride is a very sturdy material with a melting point of approximately 800℃. However, when sodium chloride is dissolved in water, the resulting solution contains high concentrations of individual Na^+ and Cl^- ions and conducts electricity readily.

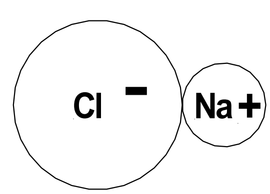


Figure 30 Sodium Chloride Molecule (source: author)

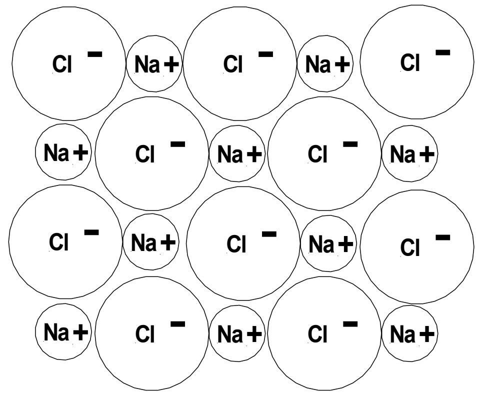


Figure 31 Solid Sodium Chloride Lattice Structure (source: author)

The Na^+ and Cl^- ions in a sodium chloride molecule form a permanent electric dipole, and associated dipole moment, within the molecule. In general, ionically bound molecules have a significant dipole moment.

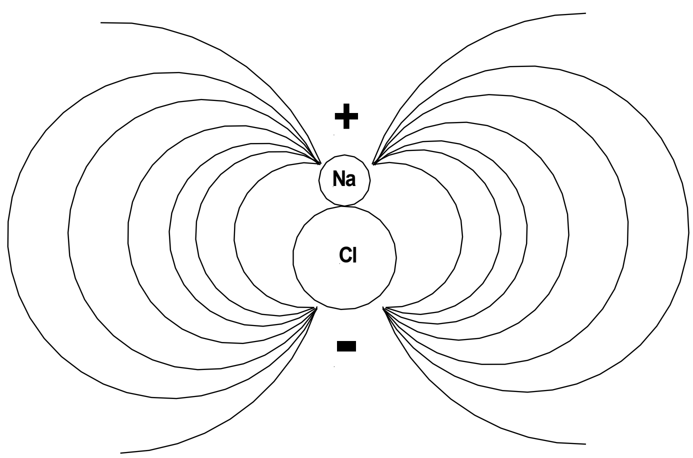


Figure 32 Sodium Chloride Electric Field (source: author)

The dipole moment is defined as

$$Dipole\ Moment\ \equiv \vec{p} = q\vec{l}$$

where q is the positive or negative charge that an atom involved in an ionic or covalent bond may possess and $\vec{l}$ is the distance between the centers of the two bonded atoms. If one of the atoms in a bond is slightly positive then the other atom will be slightly negative.

7.2 Covalent Bonds

The atoms in a molecule held together by covalent bonding share valence electrons. By sharing electrons, the energy level of the two bound atoms is lower than either atom by itself. Covalent bonds are in general weaker and more easily broken than ionic bonds.

Water H_2O is an example of a covalently bound molecule. The covalent bonds in a water molecule, shown in Figure 33 below, are formed by electron sharing between the oxygen atom and each of the hydrogen atoms. That is, the hydrogen atom electron spends part of its time in the valence shell of the oxygen atom and part of its time orbiting around the hydrogen nucleus. If the electron were shared equally between the two atoms, both would be electrically neutral because, on average, the number of electrons around each atom would equal the number of protons in that atom's nucleus. However, the electron sharing in a water molecule is not equal. The oxygen atom has a greater affinity for the shared electron than the hydrogen atom. Thus the electron spends more time in the oxygen atom valence shell than orbiting around the hydrogen nucleus. Consequently the oxygen atom gains a slight negative charge and each of the hydrogen atoms a slightly positive charge. Because of this unequal charge distribution, water is said to be a polar covalent molecule. Thus, the water molecule possesses a permanent internal dipole moment. The total dipole moment for the water molecule is equal to the vector sum of dipole moments between the oxygen atom and each of the hydrogen atoms. It is this net dipole moment that gives water its great ability to dissolve compounds.

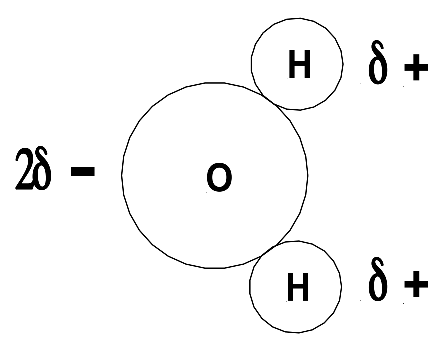


Figure 33 Water Molecule (source: author)

Other covalently bonded molecules possess internal dipole moments of various degrees from substantial to none at all. The strength of the dipole moment, if any, depends on how equally the valence electrons involved in the covalent bonds are shared between the molecule's bound atoms.

- A dipole moment will **not** form if the valence electrons are shared equally between the bound atoms.

- In contrast, a dipole moment **may** form if the electrons are **not** shared equally.

The dipole moment for a molecule as a whole is equal to the vector sum of the individual dipole moments associated with each of the molecule's covalent bonds. The vector sum will be zero, resulting in no net dipole moment, if the covalent bonds are distributed symmetrically around the molecule. For example, a molecule with a slightly negative central atom surrounded by symmetrically arranged slightly positive atoms will exhibit little or no dipole moment. The carbon dioxide molecule CO_2 , shown in Figure 34 below, has two covalent bonds each with a dipole moment. But since carbon dioxide is a symmetrical linear molecule, the two dipole moments cancel out leaving the molecule with no net dipole moment.

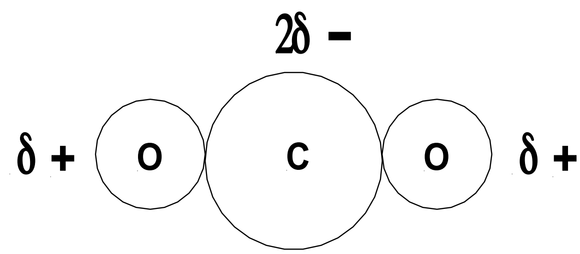


Figure 34 Carbon Dioxide Molecule (source: author)

In contrast, the hydrogen atoms in a water molecule are **not** symmetrically arranged around the oxygen atom. The non-symmetric arrangement results in the water molecule's strong dipole moment.

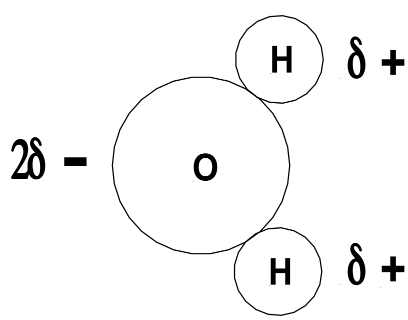


Figure 35 Water Molecule (source: author)

8 Radioactivity Reveals the Age of the Earth

In 1862 physicist William Thomson (Lord Kelvin) determined the age of Earth to be 20 to 40 million years old. His estimate was based on the time he calculated for the Earth's crust to cool from its initial molten condition to the surface temperature he was able to measure. His calculations did not account for thermal convection currents within the Earth or radioactive decay, both of which were unknown at the time.

Other estimates of the Earth's age were made by Hermann von Helmholtz and astronomer Simon Newcomb. In 1856 von Helmholtz estimated the age of the Earth to be 22 million years old. Newcomb in 1892 calculated the age to be 18 million years. Both based their estimates on the time that it would take the Sun to form from a nebula of gas and dust and then cool to its current size. They assumed the light and heat radiated by the Sun was the result of solar gravitational contraction. The process of solar nuclear fusion was unknown.

Geologists had a hard time accepting such a young age for the Earth. So did some biologists. If Darwin's theory of evolution was correct, over a billion years would be required for life on Earth to evolve to its present level. Today's estimates are that life began on Earth 3.5 to 3.8 billion years ago.

Radioactivity was initially discovered by Henri Becquerel in 1896. Marie and Pierre Curie discovered radioactive polonium and radium in 1898. Subsequently Pierre Curie and Albert Laborde showed in 1903 that radium produced heat as it decayed. These discoveries invalidated the previous estimates of Earth's age. The earlier estimates all assumed that the initial heat of both the Sun and the Earth was continuously dissipated into space at a constant rate causing both bodies to cool at a predictable rate. Discovery of radioactive decay meant that heat was continuously being generated within the Earth in addition to being radiated into outer space.

Ernest Rutherford and Frederick Soddy discovered that radioactive decay was due to an element decomposing into another lighter element (an element with fewer protons and neutrons in its nucleus), producing alpha, beta, and gamma radiation in the process. They also discovered that a radioactive element decayed into another element at a specific rate. They quantified this rate of decay as the half-life of the radioactive element. Half-life being the time its takes for half the mass of a radioactive element to decay into some other element.

As discussed earlier, all atoms of a specific element have the same number of protons. However, the number of neutrons in the nucleus can vary, within limits, from one atom to the next. All atoms with the same number of neutrons are designated as a particular isotope of the element. For example, Uranium-238 and Uranium-235 are two commonly occurring isotopes of uranium. 238 is the atomic mass of Uranium-238 (${}_{92}U^{238}$) and 235 is the atomic mass of Uranium-235 ($_{92}U^{235}$). Both isotopes have the same number of protons, 92. $_{92}U^{238}$ has 146 neutrons (238 – 92) while $_{92}U^{235}$ has 143 neutrons.

Radioactive decay occurs when the unstable nucleus of a radioactive element, such as uranium, spontaneously changes releasing energy and altering the number of protons and neutrons in the atom's nucleus. In the process the original (parent) atom becomes a different element since the number of protons in the nucleus has changed. The element that is formed by the radioactive decay is referred to as the daughter atom. The daughter element may itself be radioactive and decay into yet an other element. The decay chain continues from one radioactive element to the next until a stable non-radioactive daughter element is finally formed. For example, $_{92}U^{238}$ decays through a chain of 13 intermediate radioactive elements until finally becoming the stable lead isotope $_{82}Pb^{206}$.

All known isotopes of elements with atomic numbers greater than 82 are radioactive.

Radioactive decay generally involves one or more of the following 3 processes:

1. Alpha Decay

    - An Alpha partical, consisting of 2 protons and 2 neutrons is ejected from the nucleus.

    - The daughter element has an atomic number which is two less than the parent since 2 protons were lost.

    - The daughter element also has an atomic mass that is four less than the parent since 2 neutrons, in addition to the 2 protons, were ejected from the nucleus.

2. Beta Decay

    - A neutron loses an electron (a Beta particle) and becomes a proton.

    - The electron is ejected from the atom.

    - The daughter element has an atomic number one greater than the parent since it has one more proton.

    - The atomic mass of the daughter element is the same as the parent since the **sum** of protons and neutrons in the nucleus remains the same even though the number of each has changed.

3. Electron Capture

    - A high speed electron from outside the atom penitrates through the atom's electron cloud and into the nucleus where it is capatured by a proton.

    - The proton becomes a neutron.

    - The atomic number of the resulting daughter element is one less than the parent since a proton was changed into a neutron.

    - The atomic mass of the daughter element is the same as the parent since the **sum** of the protons and neutrons in the nucleus remains the same even though the number of each has changed.

The half-life of some radioactive elements is very short, on the order of minutes or seconds. The half-life of other elements can be extremely long. For example, the half-life of radioactive potassium ${}_{19}K^{40}$ is 1.3 billion years. Suppose that 1 mg of radioactive potassium was present in a rock when it formed. 1.3 billion years later half of the original 1 mg will have decayed into the stable isotope of argon ${}_{18}Ar^{40}$. The 1.3 billion year old rock now contain 0.5 mg of ${}_{18}Ar^{40}$ that was not originally present when the rock formed. The quantity of ${}_{19}K^{40}$ present in the rock has been deminished to 0.5 mg , half the original quantity. After an other 1.3 billion years the now 2.6 billion year old rock will contain 0.25 mg of ${}_{19}K^{40}$, half the quantity pressent at the end of the first 1.3 billion years, and 0.75 mg of ${}_{18}Ar^{40}$. After yet an other 1.3 billion years, the ${}_{19}K^{40}$ remaining in the rock will have deteriorated to 0.125 mg while the amount of ${}_{18}Ar^{40}$ will have grown to 0.875 mg.

The half-life of uranium ${}_{92}U^{238}$ is 4.5 billion years. After 4.5 billion years half of the ${}_{92}U^{238}$ originally present will have decayed into the stable isotope of lead ${}_{82}Pb^{206}$. The lead isotope ${}_{82}Pb^{206}$ can only be formed by the decay of uranium ${}_{92}U^{238}$.

Arthur Holmes and several other physicists, including Rutherford, concluded that radioactive decay could be used to date the age of rocks. The approach that was finally used consisted of comparing the amount of naturally occurring radioactive impurity in a rock, for example trace amounts of uranium, to the amount of stable decay product, in this case lead. For example, the naturally occurring mineral zircon (chemical composition $Z_r S_i O_4$) can accommodate uranium atoms (as an impurity) into its crystal structure when it is formed. Lead atoms are not compatible with the crystal structure and thus do not naturally appear in zircon. Any lead later found in zircon can only have gotten there by some of the uranium atoms decomposing into lead. Measuring the amount of uranium present, plus the amount of lead, and knowing the half life of uranium allows the age of the rock to be determined.

The equation used to determine the age t of the rock is

$$N_P = N_{P0}e^{-\gamma t}$$

where

N_P = number of atoms of a radioactive isotope ($_{92}U^{238}$ in this example) currently present

N_{P0} = number of radioactive isotope atoms originally present when the rock formed

e = 2.718

γ = the radioactive decay constant.

The number of radioactive isotope atoms originally present when the rock formed is equal to

$$N_{P0} = N_P + N_D$$

where

N_D = number of daughter atoms (stable end product plus intermediate daughter products).

The radioactive decay constant γ is calculated by the following equation

$$\gamma = \frac{\ln 2}{t_{HL}} = \frac{0.693}{t_{HL}}$$

where

t_{HL} = the half-life of the radioactive isotope.

Substituting the value of γ into the first equation gives

$$N_P = N_{P0} \cdot e^{-(0.693/t_{HL})t}$$

Then taking the natural logrithm

$$\ln N_P = -\left(\frac{0.693}{t_{HL}}\right) \cdot t \cdot \ln N_{P0}$$

Solving for the age t of the rock gives

$$t = -\left(\frac{t_{HL}}{0.693}\right) \cdot \ln \frac{N_P}{N_{P0}}$$

As an example, assume that a mass spectrometer determines that there are currently 1,500,000 atoms of uranium $_{92}U^{238}$ in a rock sample ($N_P = 1{,}500{,}000$) and that there are 300,000 atoms of lead $_{82}Pb^{206}$ and other intermediate daughter products currently present in the rock sample ($N_D = 300{,}000$) .

The original quanty of $_{92}U^{238}$ in the rock is

$$N_{P0} = N_P + N_D = 1{,}500{,}000 + 300{,}000 = 1{,}800{,}000$$

Knowing that the half-life of $_{92}U^{238}$ is 4.5 billion years

$$t_{HL} = 4.5\ (10^9)$$

The age of the rock sample is then

$$t = -\left(\frac{t_{HL}}{0.693}\right) \cdot \ln \frac{N_P}{N_{P0}} = -\left[\frac{4.5(10^9)}{6.93(10^{-1})}\right] \cdot \ln \frac{1.5(10^6)}{1.8(10^6)}$$

$$t = -[6.49(10^9)] \cdot (-0.182) = 1.18(10^9)$$

$$t = 1.18\ \ billion\ years\ old$$

Gradually the other physicists branched off into other aspects of nuclear study. Holmes continued with the research into radioactive dating. He published a number of papers both before and after World War I documenting his work. However, his papers were generally ignored.

In 1927 Holmes presented a paper in which he estimated the age of the Earth to be 1.6 to 3.0 billion years old based on his radioactive dating of rocks. However, this report was not well accepted either, particularly by the geological community who resented physicists medaling in their domain. By 1931 the success of radioactive dating had become significant enough to prompt the U.S. National Academy of Science to conduct an in depth study to determine the age of the Earth. Holmes was a member of the study committee. In fact, Holmes wrote most of the final report. The report described in great detail what had been done, how it was done, and the results achieved. The report was so detailed it achieved wide spread acceptance and established radioactive (radiometric) dating as the most reliable means for determining the age of the Earth.

Trying to determine the age of the Earth from geological rock samples is difficult because the rocks are generally not in their original form. Usually they have been modified by the actions of plate tectonics, hydrothermal circulation within the Earth, and weathering.

In 1956 Clair Cameron Patterson published a report on radiometric dating that he had performed on meteorite fragments found in the Barringer Impact Crater in Northern Arizona, a short distance from Flagstaff. The results of his study indicated that the Earth was 4.55 billion years old.

Radiometric dating of meteorites is generally accepted as being more reliable in determining the age of the Earth than dating terrestrial rocks. The reason for this is relatively simple. Meteorites are believed to have formed at about the same time as the Earth. However, in orbiting the Sun meteorites have remained in their initial pristine condition, undisturbed by the all the forces and weathering to which terrestrial rocks are subjected. Rocks on the surface of the Moon are also believed to be in their original undisturbed state. Rocks returned by the Apollo missions have been dated and found to be no more than 4.51 billion years old.

Interestingly, some of the meteorites found on Earth are actually debris ejected from the surface of Mars during meteor impacts on the planet. These Martian meteorites have also been dated to around 4.5 billion years old.

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