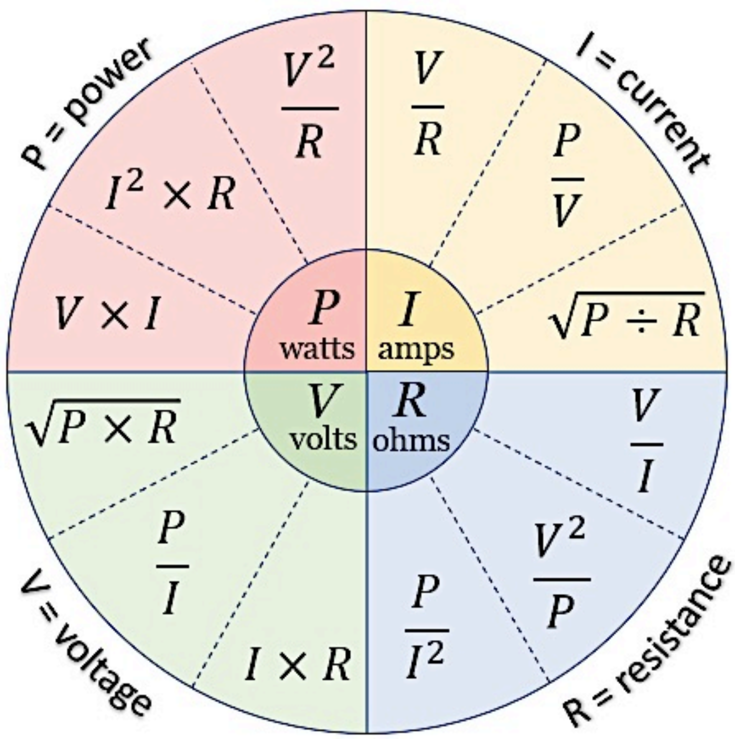
Units of Measure History



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1 Electrical and Related Mechanical Units of Measure

The International System of Units, universally known as the SI, defines the internationally accepted electrical units. These units include the ampere, ohm, volt, coulomb, faraday, and henry.

The electrical units were originally defined and agreed upon at the First International Congress of Electricians in 1881. They were reaffirmed and clarified at the Second International Congress of Electricians in 1893. Extensive research into the basis for defining the electrical units the way they were and into implementation of the international ohm, ampere, and volt continued in the years following 1893. In 1948 the electrical units were completely redefined, by the General Conference on Weights and Measures (the CGPM), to incorporate the extensive changes in technology that had occurred since 1893. While the definitions were completely changed, the changes were made in such a way that the standard value representing each unit remained approximately the same. Making changes in this manner were necessary so as not to disrupt the international electrical and electronic industries. For example, the electromotive force (emf) of the standard Weston cell (one cell of the Weston battery) was defined as 1.0183 volts prior to the 1948 change and 1.01864 volts after the change.

The SI was established and defined by the CGPM in 1960, and carried forward the electrical units as defined in 1948. SI comes from the French word System International d'Unites. Since 1960 the definitions for the electrical units have gradually changed from being based on the measurement of physical objects, such as the force between two parallel wires, to being based instead on constants of nature. Constants of nature include the speed of light, the charge of an electron, the Josephson constant, the quantum-Hall effect, Planck's constant and Boltzmann's constant. Again, the changes were carefully made so as not to disrupt industry, while at the same time re-defining the electrical units in a far more precise and invariant manner.

This appendix describes the electrical units, along with their associated mechanical units, and provides a brief history of how these units evolved and became accepted.

2 International Bureau of Weights and Measures

The International System of Units (the SI) is maintained by the International Bureau of Weights and Measures, headquartered in Sevres, France.

On May 20, 1875 an international treaty know as the Metre Convention was signed by 17 European countries, including France, Germany, Italy, Spain, and Russia. This treaty established a set of three interrelated organizations to develop and maintain a single coherent internationally agreed upon system of weights and measures. The three organizations are:

- General Conference on Weights and Measures (French abbreviation CGPM)
- International Committee for Weights and Measures (French abbreviation CIPM)
- International Bureau of Weights and Measures (French abbreviation BIPM)

The member states participating in the continued evolution and maintenance of the treaty has grown steadily from the original 17 members. The United States became one of the member states in 1878 and the U.K. followed in 1884.

2.1 General Conference on Weights and Measures (French abbreviation CGPM):

The CGPM is the governing body of the three inter-governmental organizations established by the treaty. It acts on behalf of the governments of its member states to maintain and evolve a single uniform system of measurements.

The CGPM:

- Appoints members to the International Committee for Weights and Measures (CIPM),
- Receives reports from the CIPM which it passes on to the governments and national laboratories of its member states,
- Examines and where appropriate approves proposals from the CIPM concerning changes to the International System of Units (SI) which it maintains,
- Approves the budget for the International Bureau of Weights and Measures (BIPM), and
- Decides on all major issues concerning the organization and development of the BIPM.

The CGPM meets at the International Bureau of Weights and Measures (BIPM) in Sevres, France (south-west of Paris) every four to six years.

Initially the CGPM was only concerned with the kilogram and the meter. In 1921 the scope of the treaty was extended to accommodate all physical measurements and hence all aspects of the metric system. In 1960 the 11th CGPM approved the International System of Units, generally know today as the SI system.

2.2 International Committee for Weights and Measures (French abbreviation CIPM):

The CIPM consists of 18 people selected by the CGPM from member states. Their principal task is to ensure world-wide uniformity in units of measurement by taking direct action or by submitting proposals to the CGPM.

The CIPM meets annually at the International Bureau of Weights and Measures (BIPM) in Sevres to discusses reports presented to it by its Consultative Committees. The CIPM is also responsible for issuing an Annual Report to its member states on the administrative and financial position of the International Bureau of Weights and Measures.

The CIPM establishes a number of consultative committees to assist it in its work. The consultative committees perform extensive studies and research into all aspects of the SI and issue recommendations to the CIPM. These committees are under the direct authority of the CIPM

2.3 International Bureau of Weights and Measures (French abbreviation BIPM)

The International Bureau of Weights and Measures is the permanent laboratory and world center of scientific metrology established by the Metre Convention. The facility is located at the Pavillon de Breteuil in Sevres, France. It is financed jointly by its member states and operates under the exclusive supervision of the CIPM.

The BIPM carries out measurement related research and performs standards calibrations for member states. The BIPM is the keeper of the international prototype of the kilogram. The BIPM also has an important role in maintaining accurate worldwide time of day. It analyzes and averages the official atomic time standards of member states around the world to create a single official Coordinated Universal Time (UTC).

3 International Standard of Units (SI)

The International Standard of Units (SI) is divided into **Basic** quantities and units plus **Derived** quantities and units. The Basic quantities are the fundamental quantities from which all other quantities are derived.

3.1 Basic Quantities and Units

The basic quantities used in the SI are:

- Length,
- Mass,
- Time,
- Electric current,
- Thermodynamic temperature,
- Amount of substance, and
- Luminous intensity.

The basic quantities are by convention assumed to be independent.

The corresponding basic units of the SI chosen by the CGPM are:

- Meter (unit of distance – length),
- Kilogram (unit of mass),
- Second (unit of time),
- Ampere (unit of electric current),
- Kelvin (unit of thermodynamic temperature),
- Mole (unit of amount of substance), and
- Candela (unit of luminous intensity).

3.2 Derived Quantities and Units

Derived quantities are defined in terms of the basic quantities. That is, each derived quantity is defined by an equation consisting of sums, products, and powers of specific basic quantities. The units for the SI derived quantities, as chosen by the CGPM, include:

- Newton (unit of force),
- Joule (unit of energy or work),
- Watt (unit of power),
- Ohm (unit of electric resistance),
- Volt (unit of potential difference and of electromotive force),
- Coulomb (unit of quantity of electricity),
- Farad (unit of capacitance),
- Henry (unit of electric inductance).

4 History of Mechanical Units

Electrical quantities were originally defined in terms of length, mass, and time, and still are related to these units. The units of force, work, and power apply to both mechanical and electrical systems. This section describes the evolution of these mechanical units.

4.1 Second - Unit of Time

On December 10, 1799 the French Government officially adopted the MKS (meter, kilogram, second) units of measure developed by the French Academy of Science.

A second was understood to be 1/86,400th of a day. That is,

$$1\ second = \frac{1\ Day}{86{,}400} = \frac{1\ Day}{(24\ hr/day)(60\ min/hr)(60\ sec/min)} \quad in\ 1799$$

as originally defined by astronomers at least 700 years earlier.

The British Association for the Advancement of Science established the cgs (centimeter, grams, seconds) units of measure in 1874. In the British system, the second was more rigorously defined as 1/86,400th of a mean solar day. That is,

$$1\ second = \frac{Mean\ Solar\ Day}{86{,}400} \quad in\ 1874$$

The cgs system was gradually replaced by the French MKS (meter, kilogram, second) system that was internationally adopted in 1948. The MKS system also defined a second in terms of the mean solar day, where the exact definition of mean solar day was left to astronomers. Thus

$$1\ second = \frac{Mean\ Solar\ Day}{86{,}400} \quad in\ 1948$$

By 1956 it was understood that the Earth's rotation (the mean solar day) was not sufficiently uniform to serve as the standard for time. To solve this problem, the second was defined in terms of the Earth's solar orbital period for the year 1900. A specific year had to be selected since the Earth's orbital period decreases linearly over time. In 1960 the MKS system was renamed the International Standard of Units (SI). The second was defined in the SI system as:

$$1\ second = \frac{Earth's\ year\ 1900\ Orbital\ Period}{31{,}556{,}925.9747} \qquad in\ 1960$$

In 1967 the second was more accurately defined using the caesium-133 atomic clock instead of the Earth's orbital period. Specifically, the second was defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the caesium-133 atom ground state. Thus,

1 second = 9,192,631,770 periods of the caesium – 133 atomic clock in 1967

At its 1997 meeting the International Committee for Weights and Measures (CIPM) affirmed that this definition refers to a caesium atom at rest at a temperature of 0 K. This note was added to make it clear that the definition of the SI second is based on a caesium atom unperturbed by black body radiation, that is, in an environment whose thermodynamic temperature is 0 K. The frequencies of all primary frequency standards had to be corrected for the shift due to ambient radiation, as stated at the meeting of the Consultative Committee for Time and Frequency in 1999.

4.2 Meter – Unit of Length

The Egyptian cubit, developed around 2700 BC, is one of the earliest units known for measuring length. The cubit was the length of the forearm from the elbow to the tip of the middle finger. Cubit rods, approximately 523 to 529 mm (20.6 to 20.8 inches) in length, were made to facilitate measuring. The cubit was divided into a span, palm, and finger:

- Span (one half cubit) – the length from the tip of the little finger to the tip of the thumb,
- Palm (one sixth cubit) – the width of the hand,
- Finger (one fourth palm) – the width of the middle finger.

The Egyptians also used the length of the human foot as a unit of measure in addition to units based on the arm, hand, and finger.

The Greeks and Romans inherited the foot as a unit of measure from the Egyptians. The Roman foot was approximately 296 mm in length and was divided into 12 inches, each inch approximately 24.7 mm long. Today an inch is 25.4 mm. The Romans created the mile which was defined as 1,000 paces, a pace being 5 feet in length. Thus a Roman mile was 5,000 feet long. Queen Elizabeth I change the English mile to 5280 feet (8 furlongs) during her reign from 1558 to 1603.

In 1668 English cleric and philosopher John Wilkins wrote an essay in which he proposed a decimal-based unit of length. Wilkins was very much involved in the scientific revolution of the 1600s. He was Warden of the Wadham College at Oxford University and later Master of Trinity College at Cambridge University. He was also one of the founders of The Royal Society of London for Improving Natural Knowledge. Today the society is known as the Royal Society and serves as the Academy of Science for the United Kingdom (UK). In his paper, Wilkins suggested using the length of a pendulum with a half-period of one second as the standard unit of length. The length of this pendulum is 39 1/4 inches long.

In the late 1700s there were two competing decimal based approaches for defining a new standard unit of length. The first was that using the period of a pendulum, as originally proposed by Wilkins. The second approach defined the standard length based on the meridian distance from the equator to the north pole. The meridian approach was selected as being more accurate because the force of gravity varies slightly over the surface of the Earth. This variation affects the period of a pendulum and thus the pendulum length required for a one second half-period.

In 1791 the French National Assembly accepted a proposal from the French Academy of Science to create a new decimal based standard unit of length called the meter (metre in French). The purpose of the meter was to replace the many different units of measure existing at the time with a single scale for all measurements. The Academy defined the meter to be one ten-millionth of the distance from the equator to the north pole measured along the Earth's meridian quadrant passing through Paris.

The distance from the equator to the north pole had to be determined as accurately as possible to produce a universally accepted standard of measurement. To do this, the distance from a belfry in Dunkerque, France (approximately 6 miles from the Belgian border) to the Montjuic Castle in Barcelona, Spain was measured using the existing French unit of distance. These two cities are located on the same longitude, a necessary condition for the measurement. The distance between the two cities, combined with their respective latitudes, enabled the distance from the equator to the north pole to be calculate. One ten-millionth of this distance, again using the existing French unit of measure, was defined as 1 meter.

A platinum 1 meter bar was manufactured in mid 1799 and officially accepted by the French National Assembly as the new French standard unit of length on December 10, 1799.

In the 1870's a series of international conferences were held to establish an internationally accepted decimal based system of measurement known as the metric system. The Metre Convention signed on May 20, 1875 formally adopted the metric system and established a permanent International Bureau of Weights and Measures to be located in Sevres, France (a short distance south west of Pairs). The organization was tasked with the responsibilities of maintaining the Standard Meter Bar and other metric standards, distributing copies of these standards to other nations as needed, and maintaining comparisons between the metric standards and non-metric measurement standards.

The International Bureau of Weights and Measures introduced a new more rugged one meter standard bar at the 1889 first General Conference of Weights and Measures, and changed the definition of a meter to include not only the length but also the composition of the bar. The new standard defined the meter as the distance between two lines on a standard bar composed of an alloy of 90% platinum and 10% iridium measured at the melting point of ice. Using this alloy increased the hardness of the new standard one meter bar compared to the original all platinum unit.

The meter was again redefined at the 7th General Conference on Weights and Measures on October 6, 1927 to achieve a more rigorous definition of a meter. At this conference the meter was defined as the distance, at 0^{o} C and one standard atmosphere of pressure, between the axes of the two central lines marked on an platinum-iridium bar supported on two cylinders of at least 1 cm diameter, symmetrically placed in the same horizontal plane at a distance of 571 mm from each other.

A radical change in the definition of the meter occurred at the 11th General Conference on Weights and Measures on October 14, 1960. Instead of defining the meter based on a physical artifact (the one meter platinum-iridium bar), the meter was defined instead in terms of a constant of nature. Specifically, the meter was defined as 1,650,763.73 wavelengths of the orange-red emission line in the electromagnetic spectrum of the krypton-86 atom when measured in a vacuum. The length of a meter did not change. What changed was a more precise way of defining the length.

The meter was redefined again in 1983 to achieve an even more precise definition. The 17th General Conference on Weights and Measures on October 21, 1983 defined the meter as the length of the path travelled by light in a vacuum during a time interval of

$$\frac{1}{299{,}792{,}458} \quad of\ a\ second$$

It follows that the speed of light in a vacuum is exactly 299,792,458 meters per second. That is, the speed of light

$$c_0 = 299{,}792{,}458 \text{ m/s} .$$

The helium-neon laser is the physical realization of this standard. One meter is equal to 1,579,800.298728 wavelengths of the light produced by the helium-neon laser.

All of the standards established for the meter from 1889 to present are simply more precise specifications of the original platinum 1 meter bar manufactured in 1799.

4.3 Kilogram – Unit of Mass

Newtonian physics defines mass as the property of a physical body that determines its resistance to a change in motion. This definition is adequate for everyday objects and energy levels. The theory of special relativity requires the definition of mass be altered for very small subatomic objects and for objects traveling at very high speeds approaching the speed of light.

Newton's First Law of Motion states that every object remains in its state of rest or uniform motion in a straight line unless it is acted upon by a force. In the absence of a force, the momentum (M) of an object is constant and equal to the object's mass (m) times its velocity (v), that is:

$$\overrightarrow{M} = m \cdot \overrightarrow{v}$$

Both momentum and velocity are vector quantities.

Newton's Second Law of Motion states that the acceleration (a) of an object is proportional to, and in the same direction, as the force (F) acting on the object, and inversely proportional to the object's mass (m), that is:

$$\overrightarrow{a} = \frac{\overrightarrow{F}}{m}$$

Written in its more common form:

$$\overrightarrow{F} = m \cdot \overrightarrow{a}$$

Both force and acceleration are vector quantities.

Mass is also the property of a physical body which determines the strength of its mutual gravitational attraction with other bodies in accordance with Newton's law of universal gravitation. This law states that the attractive force (F) between any two masses (m_1 and m_2) separated by a distance (r) is

$$\overrightarrow{F} = G\frac{m_1 m_2}{(\overrightarrow{r})^2}$$

where G is a universal constant.

The special theory of relativity shows that energy contributes to the mass of very small subatomic particles and also to the mass of objects traveling at very high speeds. Thus, any stationary body having a mass has an equivalent amount of energy according to Einstein's equation:

$$E = m \cdot c^2$$

where E is the energy associated with the object, m is the object's mass, and c is the speed of light. Consequently, all forms of energy resist acceleration by a force and have gravitational attraction.

The SI unit of mass is the kilogram (kg).

Weight is a force (measured in newtons, pounds, ounces, etc.) that acts against the acceleration of gravity to render an object stationary in a direction parallel to the gravitational force. That is:

$$W = m \cdot g$$

where W is the weight of an object, m is the object's mass, and g is the acceleration of gravity at the object's location. An object in free fall, for example an object in orbit around the earth, has no weight (it is weightless). An object on the moon weights less than on earth because the moon's gravity is less than that of earth. However, in all of these cases the mass (m) of the object remains the same.

In a constant gravitational field, such as on the surface of the earth, the weight of an object is proportional to its mass. In this environment the same unit of measure, the kilogram for example, is often used to indicate both an object's weight and its mass. This is not a problem as long as a precision of a few percent is adequate. However, slight variations occur in the strength of Earth's gravitational field from one location to an other on the Earth's surface. Consequently, for high precision measurements, the weight of an object must be measured in newtons and that weight converted to mass in kilograms using the acceleration of gravity at that location.

In 1795 the French Academy of Science defined the gram to be the mass of one cubic centimeter of water at 4^{o} C. The concept of defining a unit of mass in terms of a specified volume of water was originally proposed by John Wilkins in 1668.

At the time the gram was considered too small for practical use in commerce and trade. To solve this problem the kilogram (1,000 grams) was defined as the standard unit of mass. In 1799 an all platinum one kilogram standard cylinder was manufactured equal in mass to one cubic decimeter of water at 4^{o} C. The platinum standard one meter bar was also manufactured at the same time. Both physical standards were accepted by the French National Assembly on December 10, 1799.

It is interesting to note that the extremely important work of defining the meter and the kilogram was performed by the French Academy of Science in France during the French Revolution. The academy was created by Louis XIV in 1666 to encourage scientific research in France and was very active in European scientific developments throughout the 1700s and 1800s. The academy was abolished in 1793, during the early part of the French Revolution, and then reformed in 1795 with most of its original members. The academy was not restricted to scientists. In 1798 Napoleon Bonaparte was elected a member of the academy.

At the time that the meter and kilogram were being defined, Volta was developing the first battery in Italy and the Chappe brothers were beginning work installing their visual telegraph system throughout France.

The 1799 platinum kilogram cylinder remained the standard for ninety years. Signing of the Metre Convention on 20 May 1875 initiated work by the newly created International Bureau of Weights and Measures on a new standard kilogram cylinder. The new kilogram standard, know as the International Prototype Kilogram, was accepted at the 1889 first General Conference of Weights and Measures along with the new standard one meter bar.

The International Prototype Kilogram is a right-circular cylinder 39.17 millimeters in height and diameter. A right-circular cylinder geometer was used to minimize surface area. The standard is made of a platinum alloy consisting of 90% platinum and 10% iridium, the same alloy used for the standard one meter bar. Using this alloy increased the hardness of the new kilogram standard compared to the original all platinum unit. Like its all platinum predecessor, the alloy has extremely high density (nearly twice that of lead), is extremely resistant to oxidation, and has satisfactory electrical and thermal conductivities as well as low magnetic susceptibility.

The 1889 International Prototype Kilogram is stored at the International Bureau of Weights and Measures in Sevres, France.

Copies of the International Prototype Kilogram were made and distributed to countries throughout the world. These copies were compared in 1948 and again in 1989 to the International Prototype Kilogram at Sevres to insure that the copies had not deviated from the standard.

Historically the base units of the SI system have been defined by manufactured physical objects designated as the official international standards. The International Prototype Kilogram is a classic example. That has now changed. Today all of the SI base units are defined in terms of fundamental constants of nature. This change began in 1967 by redefining the second in terms of the caesium-133 atomic clock. The meter was redefined in terms of the speed of light in 1983.

In 2014 the 25th General Conference of Weights and Measures redefined the kilogram in terms of the Planck constant. The kilogram is now defined as follows:

The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant h to be exactly $6.62606957 \cdot 10^{-34}$ when it is expressed in the SI unit for action

$$J \cdot s = kg \cdot m^2/s.$$

That is

$$h \equiv 6.62606957 \cdot 10^{-34}\, joule\ second = 6.62606957 \cdot 10^{-34}\, \frac{kg\, m^2}{s}$$

The new definition for the kilogram (mass) is thus dependant on the definition of both time and length (seconds and meters).

The numerical value of the Planck constant is chosen so that the new definition for the kilogram is equal to the mass of the International Prototype Kilogram within a few parts in 10^8. A few parts in 10^8 is the uncertainty of the combined best estimates of the value for the Planck constant at the time of the 25th CGPM.

4.4 Kelvin – Unit of Temperature

Swedish astronomer Anders Celsius developed a temperature scale in 1742 in which the boiling point of water was defined as 0 degrees with 100 degrees being water's freezing point, the reverse of what is used today. Celsius recognized that the boiling point of water varies with atmospheric pressure. To account for this, he defined the zero point on his scale as being the boiling point of water at sea level (at one standard atmosphere of pressure).

In 1743 French physicist Jean-Pierre Christin, unaware of the work done by Celsius, develop a temperature scale in which the freezing point of water was defined as 0 degrees and 100 degrees represented the boiling point of water. On May 19, 1743 he published the design of a mercury thermometer that used his temperature scale.

In 1744, the same year that Celsius died, Swedish botanist Carolus Linnaeus reversed the scale developed by Celsius. Linnaeus assigned 0 degrees to the freezing point of water and 100 degrees as water's boiling point. Linnaeus was unaware that Christin had developed the same scale the year before.

The temperature scales independently developed by Christin and Linnaeus, with 0 degrees being the freezing point of water and 100 degrees water's boiling point, became known as the centigrade temperature scale, that is ^{o}C. The term centigrade signified 100 steps between the freezing and boiling points of water. The temperature scale retained this name from 1744 to 1948. The 9th General Conference on Weights and Measures in 1948 changed the name of the temperature scale from centigrade to Celsius. This was done because centigrade, meaning 100 divisions, was used by various countries in referring to units of measure other than temperature. Renaming the temperature scale to Celsius eliminated that problem.

In 1848 Glasgow University engineer and physicist William Thomson (Lord Kelvin) published a paper titled "On an Absolute Thermometric Scale". In this paper Thomson described the need for a temperature scale in which the scale's zero point was the coldest possible temperature, that is, the temperature at which all thermodynamic motion stops. Thomson calculated that this temperature point was -273 ^{o}C based on the expansion coefficient of gas per degree centigrade relative to the melting point of ice. Thomson proposed that the incremental unit on this new scale be the same as that for the centigrade scale.

The 10th General Conference on Weights and Measures in 1954 establish the kelvin (K) temperature scale. The conference defined the zero point of this scale to be

$$\frac{1}{273.16}$$

of the thermodynamic temperature of the triple point of water. The incremental units of the kelvin scale were defined to be the same as the Celsius scale. Thus, the kelvin scale is defined such that the triple point of water occurs at exactly 273.16 K.

The triple point of water is the temperature and pressure at which solid ice, liquid water, and water vapor can coexist in stable equilibrium. This point occurs at exactly 0.01 ^{o}C (273.16 K) and a partial vapor pressure of 611.73 pascals (6.1173 millibars or 0.0060373 atm). The solid-liquid-gas triple point of water is the lowest pressure at which liquid water can exist. At pressures below the triple point, for example in mountainous areas, solid ice when heated is converted directly into water vapor, a process know as sublimation. At pressures above the triple point, solid ice when heated first melts to form liquid water and then evaporates as water vapor.

In 2014 the 25th General Conference of Weights and Measures redefined the kelvin in terms of the Boltzmann constant k. The kelvin is now defined as follows:

The kelvin, symbol K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant k to be exactly 1.3806488 (10^{-23}) when it is expressed in the SI unit for energy per thermodynamic temperature

$$J \cdot (K^{-1}) = kg \cdot m^2 \cdot (s^{-2})(K^{-1})$$

That is

$$k = 1.3806488(10^{-23})\frac{J}{K} = 1.3806488(10^{-23})\frac{kg \cdot m^2}{s^2 \cdot K}$$

The effect of this definition is that the kelvin is equal to the change of thermodynamic temperature that results in a change of thermal energy kT by $1.3806488(10^{-23})$ joules (J).

The previous definition of the kelvin was based on an exact value assigned to the triple point of water, specifically $T_{TPW} = 273.16$ K. Because the new definition of the kelvin fixes the value of Boltzmann constant k instead of the triple point of water T_{TPW}, T_{TPW} must now be determined experimentally. The Boltzmann constant was selected so that T_{TPW} is equal to 273.16 K (its previous value), with a relative standard uncertainty of less than $1 \cdot (10^{-6})$ based on measurements of k made prior to the redefinition.

4.5 Newton – Unit of Force

The SI unit of force is the newton, symbol N, named in recognition of Isaac Newton's work in classical mechanics. The unit of force is a derived unit. The base units relevant to force are:

- the meter, unit of length, symbol m,
- the kilogram, unit of mass, symbol kg
- the second, unit of time, symbol s.

Force (F) is defined as the rate of change of momentum:

$$\vec{F} = \frac{d(m \cdot \vec{v})}{dt}$$

where force (F) and velocity (v) are both vector quantities.

As an example, this definition of force applies to rocket launch vehicles in which both the mass of the vehicle as well as its velocity change quickly following lift-off. Most of the vehicle's mass consists of fuel which decreases rapidly as the fuel is consumed by the rocket engines.

If mass is constant, then force is equal to mass times acceleration.

$$\vec{F} = m \cdot \vec{a}$$

where force (F) and acceleration (a) are both vector quantities.

In 1946 the General Conference on Weights and Measures (CGPM) standardized the unit of force in the MKS (meter, kilogram, second) system of units to be the force needed to accelerate 1 kilogram of mass at the rate of 1 meter per second squared. The 9th CGPM, held in 1948 adopted the name newton as the unit of force.

The newton as the unit of force was retained when the SI system of units was formed from the MKS system in 1960.

In the SI system, an upper case symbol (N for newton, for example) is used for a unit whose name is the proper name of a person. However, when the unit is spelled out in

English, it should always begin with a lower case letter (newton), except in situations where any word in that position would be capitalized, for example at the beginning of a sentence.

4.6 Joule – Unit of Energy or Work

The SI unit of work is the joule, symbol J, named after the English physicist James Prescott Joule who made fundamental contributions to the study of heat and energy. The unit of work is a derived unit. The base units relevant to work are:

- the meter, unit of length, symbol m,
- the kilogram, unit of mass, symbol kg
- the second, unit of time, symbol s
- the ampere, unit of electrical current, symbol A.

One joule is defined as the work expended by a force of one newton through a distance of one meter. Mathematically:

$$Work = \oint_C (\vec{F} \cdot d\vec{l})$$

where force (F) and distance (l) are both vectors. Thus work, a scalar quantity, is the vector dot product of force and the incremental unit of distance **dl** integrated over the path C along which the work is done.

One joule can also be defined as the work required to move an electric charge of one coulomb through an electrical potential difference of one volt. Mathematically:

$$(Work)_{12} = q \int_{P_1}^{P_2} (\vec{E} \cdot d\vec{l}) = qV_{12}$$

where $(Work)_{12}$ is the work in joules to move a charge of q coulombs from Point 1 (P_1) to Point 2 (P_2) through an electric field E, and V_{12} is the electrical potential difference (voltage) between points (P_1) and (P_2).

The joule was initially defined and named at the 1893 Second International Congress of Electricians held in Chicago during the Chicago World's Fair.

4.7 Watt – Unit of Power

The SI unit of power is the watt, symbol W, named in honor of James Watt, the eighteenth-century developer of the steam engine. The unit of power is a derived unit. The base units relevant to power are:

- the meter, unit of length, symbol m,
- the kilogram, unit of mass, symbol kg
- the second, unit of time, symbol s
- the ampere, unit of electrical current, symbol A.

Power (P) is the rate of doing work. One watt is defined as one joule per second. Mathematically average power is:

$$P_{avg} = \frac{\Delta\, Work}{\Delta\, t}$$

and instantaneous power is:

$$P = \frac{d(Work)}{dt}$$

Electrical power is equal to:

$$P = \frac{d(qV)}{dt}$$

where q is electrical charge and V is voltage. If voltage is constant than power is:

$$P = \frac{d(qV)}{dt} = \frac{V\, dq}{dt} = VI$$

where I is the electrical current.

The watt was initially defined and named at the 1893 Second International Congress of Electricians held in Chicago during the Chicago World's Fair.

5 History of Electrical Units

Mechanical units of measure had evolved slowly over hundreds, even thousands of years. Electrical units of measure did not have this luxury of time. The initial electrical units were quickly defined while scientists were still in the process of studying the electrical phenomena for which the units were being developed. Consequently, multiple conflicting standards were created. Those standards were replaced with newer standards which were again replaced as scientists and engineers acquired an understanding of the various electrical phenomena and the means to measure them.

The French developed the meter and gram units of measure in the mid 1790s. The meter was defined to be one ten-millionth of the distance from the equator to the north pole. The gram was defined to be the mass of one cubic centimeter of water at 4^{o} C. However, the gram turned out to be too small of a unit for practical every day commerce. So the French replaced the gram with the kilogram (1,000 grams) to create the MKS (meter, kilogram, second) system. In 1799 the French formally adopted the MKS system as their standard units for mechanical measurements. This work by the French had a significant impact on the evolution of the electrical units.

In 1832 German mathematician Carl Friedrich Gauss proposed a system of absolute units based on length, mass, and time. The units selected by Gauss in his proposal were the millimeter, milligram, and second. The work of Gauss in 1833 and later Weber in 1851 suggested that electric and magnetic quantities could be defined in terms of these three mechanical units (length, mass, and time). Weber's view was that making electrical units consistent with those used in other branches of science and engineering was highly desirable.

The British Association for the Advancement of Science became very involved in developing electrical units of measurement. Electricity was only one of the many scientific fields studied by the association. Other fields of study included mathematics, physics, astronomy, mechanics, chemistry, geology, and all aspects of biology. The British Association was thus intently focused on all fields of science. Membership in the association included James Clerk Maxwell, William Thomson (Lord Kelvin), and other top British scientist.

The British Association established a committee on electrical units and standards in 1861 with Maxwell as chairman. The committee recognized the value of basing the electrical quantities on standard mechanical units as suggested by Gauss and Weber. In 1863 the committee recommended that electrical quantities be defined in terms of decimal multiples of the cgs mechanical units of centimeter, grams, and seconds.

In 1874 the British Association for the Advancement of Science formally introduced their cgs (centimeter, gram, second) system which included their proposed electrical and magnetic units. This recommendation lead to very complex definitions for electrical and magnetic units.

Conversion between the French MKS and British cgs mechanical units is simple and straight forward. For example, a meter equals 100 centimeters. A kilogram equals 1,000 grams, and the second is the same in both the French and English system. The cgs electrical and magnetic units, however, are not straight forward.

At various points in time there were over a half dozen systems of electrostatic and electromagnetic units in use. Nearly all of these were based on the cgs system. The most important of the competing cgs electrical and magnetic units were:

- Electro-Static Units (es),
- Electro-Magnetic Units (em),
- Gaussian Units, and
- Heaviside-Lorentz Units.

Each of these system of units has it own set of equations describing the physical laws of electrostatics and electromagnetics. The equations from one system to another differ because of the way the units in each system are defined. For example, there are several different versions of Maxwell's equations. The particular version used depends on the system of units selected. The conversion from one set of cgs electrical units to another is very complex.

The French and Germans were also developing electrical units. The French measured resistance in kilometers. The French resistance standard consisted of an iron telegraph wire 4 millimeters in diameter and one thousand meters long. The Germans used the Siemens unit of resistance which is defined as the resistance of a mercury column one meter long and one square millimeter in cross section (Figure 1). The French and Germans, as well as the English, used the Daniell cell as their unit of reference for emf (Figure 2).

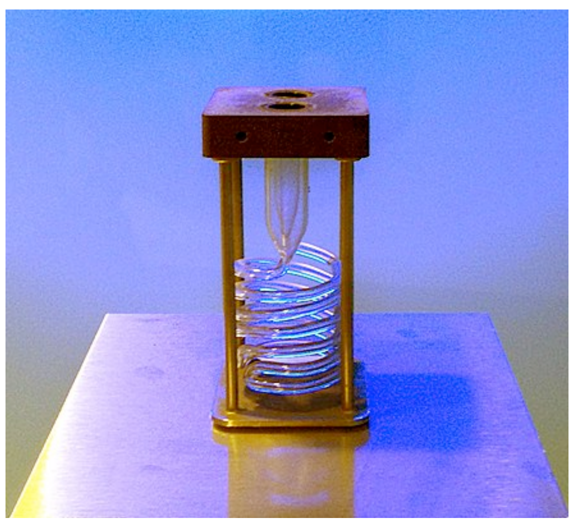


Figure 1 Siemens Unit of Resistance (source: Wikipedia)

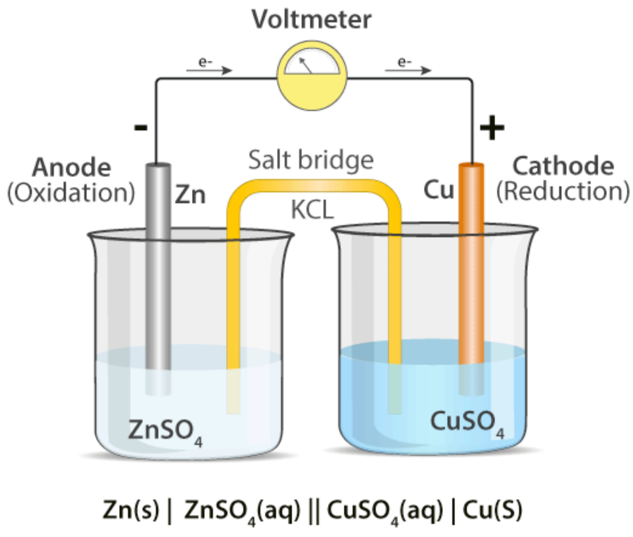


Figure 2 Daniell Cell (source: Wikipedia)

5.1 1881 First International Congress of Electricians - Paris

The 1st International Congress of Electricians met in September 1881 to establish a single set of universally accepted electrical units of measure. The meeting occurred in conjunction with the 1st International Exhibition on Electricity being held in Paris. 250 delegates from 28 different countries attend the congress. These attendees included the top scientist of the time.

The following decisions were made during the 1881 International Congress of Electricians:

1. Electrical units would be named after the scientists who made significant contributions to the understanding and practicle use of electricity.

2. The cgs Electro-Magnetic system (cgs-em), proposed by the British Association for the Advancement of Science, was adopted as the standard system for electrical measurement. In accepting the British system, the congress agreed that defining electrical quantities in terms of decimal multiples of the cgs mechanical units (centimeter, grams, and seconds) was highly desirable and would be the approach used. Decimal multiples meant that only multiples in the form $1 \cdot (10^n)$ would be used, where n is an appropriate integer. The integer n was, in each case, selected so that the fundamental (theoretical) set of electrical units being established by the congress would match as close as possible the ohm, volt, and ampere physical standards being used by engineers and electricians. To put this in perspective, at the time of the congress in 1881 a highly successful, and profitable, multi-national world wide telegraph system was in operation. This

telegraph system interconnected parts of Europe, Russia, the U.S., Canada, South America, the middle east, India, Indonesia, Chine, and Australia. In addition, factories and power generation stations using newly developed electrical machinery were being built in support of the industrial revolution. The congress was not free to set up whatever standards they wanted. In establishing a set of rigorous scientifically defined electrical units, they had to adhere as closely as possible to the ad hock electrical unit already in use by industry.

3. Resistance: The unit of resistance was defined as the ohm in honor of Georg Ohm for his work in developing Ohm's Law. The value of the ohm was set at:

$$1\ ohm = (10^9)\left[\frac{cm}{s}\right]$$

The factor 10^9 was select so that the Siemens mercury column resistance standard widely used throughout Europe would have a value of approximately 1 ohm. Its actual value was about 0.94 ohms.

4. Voltage: The unit of emf was defined to be the volt in honor of Alessandro Volta for his invention of the battery. The value for a volt was set at:

$$1\ volt = (10^8)\left(\frac{g^{1/2}\ cm^{3/2}}{s^2}\right)$$

The factor 10^8 was selected so that the emf of a Daniell cell would be approximately 1 volt. Its actual value was approximately 1.10 volts.

5. Current: The unit of electrical current was defined as the Ampere in honor of Andre-Marie Ampere for his work in defining Ampere's Law.

The value of one ampere was defined through Ohms law as the current produced by placing a 1 ohm resistance across an emf of 1 volt. This corresponded to

$$1\ ampere = 0.1\left(\frac{g^{1/2}\ cm^{1/2}}{s}\right)$$

6. Charge: The unit of electrical charged was defined as the coulomb in honor of Charles-Augustin de Coulomb for his work in defining Coulomb's Law.

One coulomb of charged was defined as the quantity of electricity produced by a current of one ampere for one second.

7. Capacitance: The unit of capacity was defined as the farad in honor of Michael Faraday. The farad (C) was defined as the charge (Q) stored by a capacitor divided by the voltage (V) between the capacitor plates. That is

$$C = \frac{Q}{V}$$

5.2 1893 Second International Congress of Electricians - Chicago

The second official International Congress of Electricians was held August 21–25, 1893 as part of the Chicago World's Fair. The following decisions were made:

1. Agreements reached at the congress will have the force of international law.

2. Electrical units already chosen were confirmed and clarified.

3. Resistance: The fundament unit of ohm was clarified as:

   - $1\ ohm = (10^9)\left(\frac{cm}{s}\right)$

   - The physical standard for 1 International Ohm was clarified as a column of mercury with a cross section of one square millimeter, 106.3 cm in length, and a mass of 14.4521 grams at 0^o C (the temperature of melting ice).

4. Ampere: The fundamental unit of ampere was clarified as:

   - $1\ ampere = 0.1\left(\frac{g^{1/2}\ cm^{1/2}}{s}\right)$

   - The physical standard for 1 International Ampere was clarified as the current that deposited 0.001118 grams of silver per second on the cathode of a silver nitrate electrolyser (silver coulometer), Figure 3.

5. Voltage: The fundamental unit of volt was clarified as:

   - The electromotive force that, steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere.

   - The physical standard for 1 International Volt was clarified as the electromotive force corresponding to 1,000 / 1,434 of the voltage produced by a Clark cell.

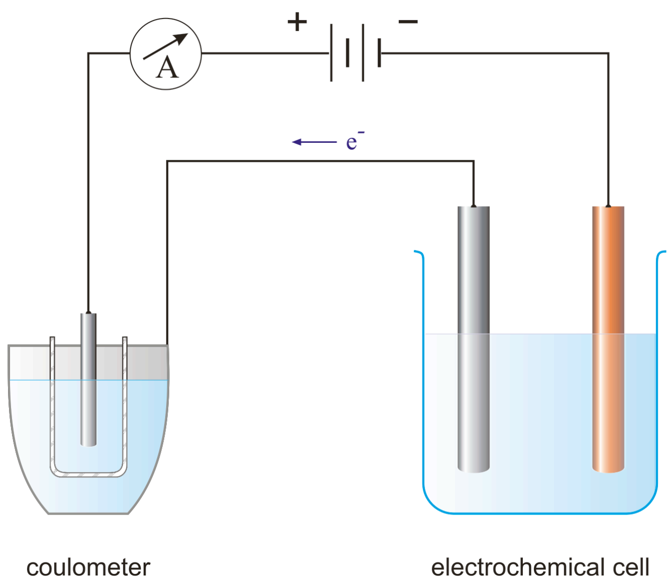
   - The Clark cell, which produced a very stable emf output, replaced the Daniell cell as the standard voltage reference, Figure 4.

6. Charge: The coulomb was clarified as the quantity of electricity transferred by a current of one ampere in one second.

7. Capacitance: The farad was clarified as the capacity of a condenser charged to a potential of one volt by one coulomb of electricity.

8. Inductance: The unit of magnetic inductance was defined as the henry, named in honor of Joseph Henry for his work in magnetic induction and development of practical electro-magnets.

   The henry was defined as the induction in a circuit when the electromotive force induced in this circuit is one volt, while the inducing current varies at a rate of one ampere per second



A Silver coulometer consists of a platinum vessel which acts as a cathode and contains a solution of pure silver nitrate. A rod of pure silver enclosed in a porous pot acts as the anode. After electrolysis, the electrolyte is taken out and the platinum vessel is washed, dried and weighed. The increase in the weight gives the amount of silver deposited.

Figure 3 Silver Coulometer (source: periodni.com)

The Chicago congress reaffirmed the definitions established by the 1881 congress for the ohm, ampere, and volt based on the cgs units of length, mass, and time. The Chicago congress also saw the utility in specifying reproducible physical standard implementations for realizing the three units. The congress formally recommended the legal adoption of both approaches as being mutually equivalent.

The original Clark Cell was a glass jar similar to a gravity Daniell Cell. The copper cathode was replaced by a pool of mercury at the bottom of the jar. Above this was a mercurous sulfate paste and above that, the zinc sulfate solution. A short zinc rod dipped into the zinc sulfate solution. The zinc rod was supported by a cork with two holes, one for the zinc rod and the other for a glass tube reaching to the bottom of the cell. A platinum wire, fused into the glass tube, made contact with the mercury pool. When complete, the cell was sealed with a layer of marine glue.

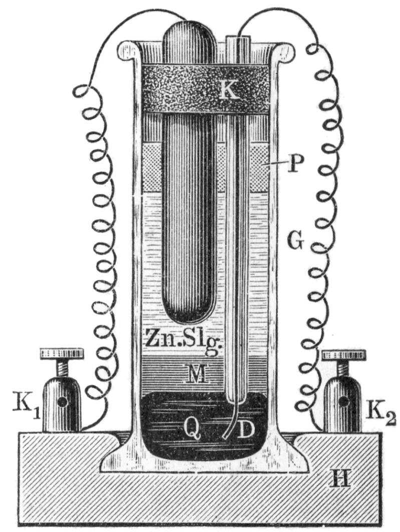


Figure 4 Clark Cell (source: Wikipedia)

The International Conference on Electrical Units and Standards held in London in October 1908 adopted the nomenclature "fundamental" as the definitions for the ohm, ampere, and volt based on the cgs units. They adopted the terms International Ohm, International Ampere, and International Volt as the physical standard implementations of the three units. For simplicity and clarity, the 1908 nomenclature is used in the above definitions for ohm, ampere, and volt.

On July 12, 1894 the United States Congress passed Public Bill No. 105 making the recommendations of the 1893 International Congress of Electricians the legal units of electrical measurements in the United States.

5.3 1904 International Electrical Congress – St. Louis

The cgs Electrostatic (es) and the cgs Electromagnetic (em) units were both still in common use in 1904. There was a strong desire within the electrical community to standardize on a single unit of measure. It was not simply a case of officially selecting either the es or the em system of units. Both systems had problems. While the em units were ideal for working with electro-magnetic systems, they was not very convent for solving electro-static problems. The reverse was also true. The challenge was to come up with a new standard system of units for electrical measurement that would be easy to use in solving both electro-magnetic and electro-static problems. The situation was further complicated by the fact that the practical units of electrical measurement (the ampere, ohm, and volt) had legal significants in daily commerce. While a new universal system of electrical units might be acceptable in the scientific community, it certainly would not be accepted by the business world if the definition of the ampere, ohm, and volt were changed.

In 1901 Italian electrical engineer and professor Giovanni Giorgi suggested a solution for reconciling the two cgs electrical systems in a manner that he hoped would be acceptable to both the scientific and business communities. Giorgi's approached had been discussed at various electrical conferences in Italy, England, and Germany in the three years leading up to the 1904 International Electrical Congress. It was the belief of many that Giorgi's approach should be seriously considered by the international delegates attending the electrical congress.

Giorgi pointed out that a fourth fundamental or independent unit was necessary, and in fact was being used, to derive electrical and magnetic units from the mechanical units of length, mass, and time. The fourth unit being used by the es system was the electrostatic constant of free space (ε_0 electric permittivity). The fourth unit used by the em system was the magnetic constant of free space (μ_0 magnetic permeability). Giorgi suggested instead that the fourth unit be one of the three electrical units ohms, ampere, or voltage. Which every unit was selected, it would have the same stature in deriving all other electrical units as the meter, kilogram and second.

Giorgi used the meter to illustrate his point. Originally the meter had been defined as one ten-millionth of the distance from the equator to the north pole. In 1799 the French manufactured a standard one meter bar equal in length to that distance. By 1900, a little over one hundred years later, the standard one meter bar had become the official definition of the meter, not the distance from the equator to the north pole. If fact, the distance from the equator to the north pole was now measured in terms of the standard one meter bar, the reverse of the original definition. Giorgi pointed out that the electrostatic constant of free space and the magnetic constant of free space were both no more important than the distance from the equator to the north pole. Using one of the practical units of electrical measurement (the ohm, ampere, or volt) was just as valid as using either the electrostatic or magnetic constants. Using one of the electrical units as the 4th fundamental unit had the additional advantage of eliminating fractional powers of length, mass, and time in defining the electrical units.

Giorgi's original preference was to use the ohm as the fourth fundamental unit. The ohm, like the meter, was defined by a physical standard. In the case of the ohm, one ohm equaled the resistance of a column of mercury with a cross section of one square millimeter, 106.3 cm in length, and a mass of 14.4521 grams at 0^{o} C. However, the ampere was eventually defined as the 4th unit instead of the ohm.

Utilizing Giorgi's approach, the electrostatic constant (the permittivity of vacuum) and magnetic constant (the permeability of vacuum) both became derived constants with the following values:

- Permittivity of vacuum = $\varepsilon_0 = 8.854 \cdot (10^{-12})$ farad per meter

- Permeability of vacuum = $\mu_0 = 4\pi(10^{-7}) = 1.256 \cdot (10^{-6})$ henry per meter

- Where

$$\sqrt{\frac{1}{\varepsilon_0 \mu_0}} = c_0 = \text{the speed of light in a vacuum}$$

the values currently used today.

Giorgi recommend use of the MKS (meter, kilogram, second) mechanical units, instead of the cgs system, since the MKS units were widely used in daily commerce. The units of centimeter and grams were considered too small for practical use in business. Giorgi's approach became known as the Giorgi MKSA (meter, kilogram, second, ampere) system.

The cgs system of electrical units (approved by the 1893 International Congress of Electricians and based on fractional powers of length, mass, and time) was finally replaced in 1948 when the 9th General Conference on Weights and Measures (CGPM) adopted the Giorgi MKSA system. The Giorgi MKSA system is considered the precursor of today's SI system of units.

5.4 National Standards Laboratories

National standards laboratories have been established by most of the world's industrialized nations. The purpose of a national laboratory is to maintain the legal units of weights and measure for its country. It is the responsibility of each laboratory to ensure that its units are consistent with internationally agreed upon units of weights and measures. Consequently, representatives from the national laboratories work with each other and the various international standards organizations, including the International Bureau of Weights and Measures, to maintain and improve international standards. Accomplishing this involves extensive research activity and frequent international meetings.

The national laboratory for the United States is the National Bureau of Standards (NBS) established by Congress in 1901 as a non-regulatory agency of the United States Department of Commerce. The National Bureau of Standards became the National Institute of Science and Technology (NIST) in 1988. Resistance, electrical current, and voltage measurements used by the electrical and electronic industries within the United States are calibrated to the United States legal ohm and volt maintained by the NBS.

Three other important national laboratories established at about the same time as the NBS are the German Physikalish-Technische Reichsanstalt (PTR) founded in 1887, the English National Physical Laboratory (NPL) founded in 1899, and the French Laboratoire National de Metrologie et d'essus (LNE) founded in 1901 .

5.5 Constraints of Ohm's Law

Ohm's law, $E = I \cdot R$ requires that one of the three units volts (E), current (I), or resistance (R) be dependant on the other two. The question of which unit was to be the dependant unit was vigorously debated in the years following the1893 Second International Congress of Electricians in Chicago. Preliminary discussions on this issue occurred during the 1904 International Electrical Congress held in St. Louis and a subsequent 1905 meeting in Berlin. A resolution drawn up at the International Conference on Electrical Units and Standards, held in London in October 1908, distinguished clearly between:

1. The theoretical "fundamental units" of ohm, ampere, volt, etc. which are exact decimal multiples of the cgs mechanical units (length, mass, and time). The fundamental units were established by the First International Congress of Electricians in 1881, and confirmed by the 1893 Second International Congress of Electricians.

2. A second system of working units (also identified by the 1893 conference) that represent the fundamental units for the purpose of routine electrical measurements. The 1893 conference believed that this second system of units was sufficiently close in value to the fundamental units that the two system of units could be used interchangeably. The London conference defined this second system of units as the International Ohm, the International Ampere, and the International Volt. As measurement technology improved, it became clear that the two systems were not exactly equal. Consequently, small changes were periodically required in the methods for implementing the second system of units to keep them as close in value to the theoretical fundamental units as possible.

The International Ohm was defined as a column of mercury with a cross section of one square millimeter, 106.3 cm in length, and a mass of 14.4521 grams at 0^{o} C, as specified by the 1893 conference. The International Ampere was defined as the current that deposited 0.001118 grams of silver per second on the cathode of a silver coulometer, also as specified by the 1893 conference. The London conference defined the International Volt to be "the electrical pressure which, when steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere." The volt thus became the derived unit in Ohm's Law while the ohm and ampere were established as the primary units. That is, the value of the International Volt was derived through Ohm's Law from the International Ohm and the International Ampere.

As the result of these decisions, the standard cells (the Daniell, Clark, and Weston cells) became secondary standards, the emf of which were determined through Ohm's Law using the International Ohm and the International Ampere.

The following reason was given for selecting the ohm and the ampere as the primary units and the derived unit being the volt. Resistors were being built that had exactly 1 ohm of resistance as defined by the cgs standards adopted in 1893. In addition, exactly 1 ampere of current could be measured using a silver coulometer. However, it was not possible with the technology at the time to build an electric cell with an emf output of exactly 1 volt. Thus the volt was selected as the dependant unit. However, the existing Daniell, Clark, and Weston cells could be calibrated using the International Ohm in combination with current measured using a silver coulometer. This was done with the following results:

- Daniell cell = 1.10 International Volts,
- Clark cell = 1.434 International Volts, and
- Weston cell = 1.0183 International Volts.

Once calibrated, these cells were used as secondary standards.

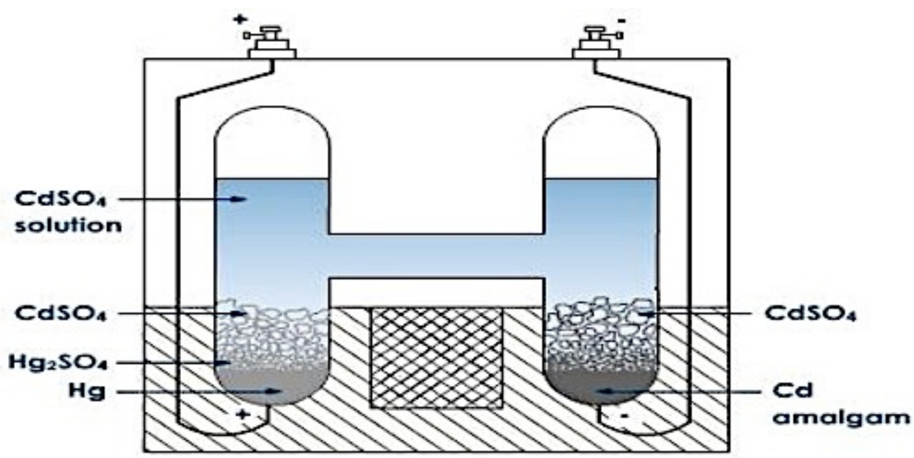


Figure 5 Weston Cell (source:qsstudy.com)

5.6 Measuring Current Based On The International Ampere

According to Faraday's law of electrolysis, a strict proportionality exists between the electrical current passing through an electrolyte and the electrochemical reaction produced by it. The electrochemical reaction can thus be used to measure the current flowing through the electrolyte. The instrument used for this measurement is a coulometer, also known as a voltameter (Figure 6).

The 1893 International Electrical Congress used a silver coulometer in specifying the International Ampere. Specifically, the International Ampere was defined as the unvarying current that deposited 0.001118 grams of silver per second on the cathode of a silver nitrate electrolyser, the electrolyser being a silver coulometer.

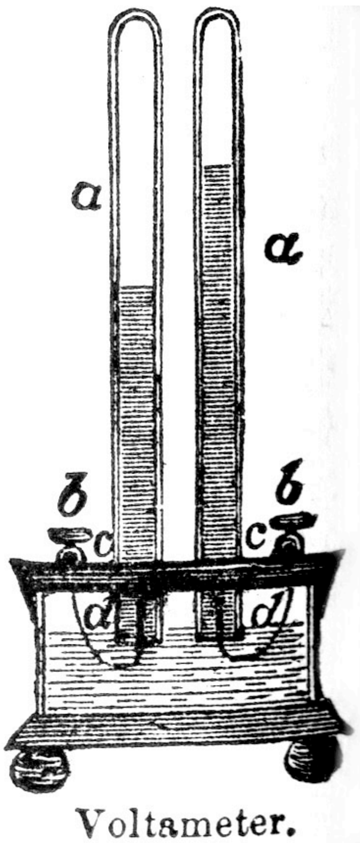


Figure 6 Voltameter also known as a silver coulometer (source: Wikipedia)

Several different types of silver coulometers where in use at the time. The Richards porous-cup coulometer, illustrated below, was one of the better designs. This coulometer consisted of a platinum pot which acted as a cathode and contained a solution of pure silver nitrate as an electrolyte $(AgNO_3)$. A rod of pure silver inside the porous cup served as the anode. An external battery, switch, and a series resistor were connected to the coulometer as shown below. It was recommended that the series resistor be at least 10 ohms so that the internal resistance of the coulometer would be negligible in comparison. The coulometer was placed into operation by closing the switch. The electrical current flowing through the coulometer caused positively charged silver ions from the anode rod to be deposited on the inner surface of the negatively charged platinum pot. The silver rod slowly disintegrated in the process. The platinum pot was weighted before and after the test. The difference in weight was equal to the amount of silver deposited. The current flowing through the coulometer was determined by the length of time the current had been flowing and the amount of silver deposited. For example, one ampere of current flowed through the coulometer if 0.001118 grams of silver were deposited on the platinum pot in one second.

One of the problems with the silver coulometer was that tinny bits of silver would separate from the disintegrating silver rod, forming a soft silver slime in the silver nitrate solution. Erroneous results would be obtained if this slime were allowed to come in contact with and adhere to the platinum pot. The porous cup prevented the slime from reaching the pot while allowing the silver ions to flow freely through the cup's pours and deposit out on the pot.

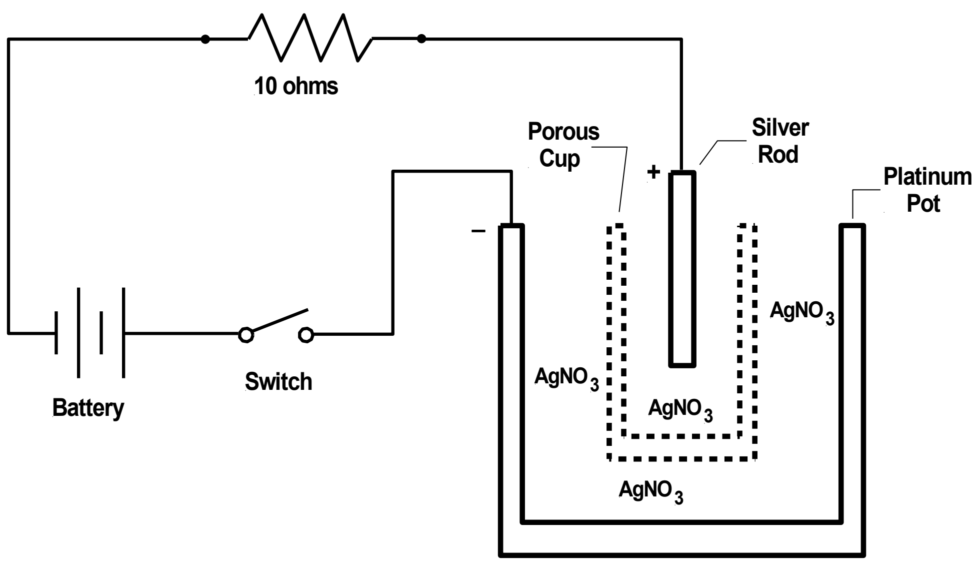


Figure 7 Richards Porous-cup Coulometer (source: author)

The process used for making an electric current measurement is as follows.

Wash the platinum pot consecutively with nitric acid, distilled water, and absolute alcohol. Dry the pot at 160°C and let it cool in a desiccator. Carefully weigh the pot after it is thoroughly cooled.

Fill the platinum pot (the cathode) with the silver nitrate solution and connect the pot to the electrical circuit.

Mount the porous cup in the silver nitrate solution. Immerse the silver rod (anode) in the porous cup and connect it to the electrical circuit.

Noting the time, close the switch to begin the measurement. The current should be allowed to flow for at least a half hour. Open the switch at the conclusion of the measurement, again noting the time.

Remove the silver rod, porous cup, and the solution from the platinum pot. Wash the pot, which now contains silver deposits on its inner surface, with distilled water and let it soak for at least six hours. After soaking, rinse the pot successively with distilled water and absolute alcohol and dry the pot in a hot air bath at a temperature of about $160^{o}C$. After cooling in a desiccator, weigh the platinum pot again. The difference in weight is the mass of the deposited silver.

To find the average current in amperes, the deposited mass (D) expressed in grams must be divided by the number of seconds (T) during which the current flowed and by 0.001118. That is,

$$I = \frac{D}{T} \cdot \frac{1}{0.001118} \ \ ampere$$

This is a very laborious method of measuring current. However, it is a suitable process for accurately calibrating standard cells. The emf of the battery in the above procedure was not specified. The battery emf is equal to the produce of the measured current and the circuit's external resistance. This process was used to calibrate the Clark and Weston standard cells.

5.7 The United States Standard Ohm

From 1901 through 1990, highly stable precision wire wound resistors have been used by the United States National Bureau of Standards (NBS) as the U.S. standard ohm. These resistors were periodically calibrated against the mercury standard International Ohm maintained in both England and Germany. The International Ohm was redefined in 1990 based on the quantum Hall effect.

5.7.1 Mercury Ohm

The mercury ohm was developed in Germany by Werner von Siemens in 1860. A version of the Siemens mercury ohm was adopted as the standard International Ohm by the 1893 International Electrical Congress. The mercury ohm (a column of mercury with a cross section of one square millimeter, 106.3 cm in length, and a mass of 14.4521 grams at $0^{o}C$) remained the official standard for the International Ohm until 1948.

Mercury was chosen in constructing the standard ohm because it could be easily purified, had a relatively high resistivity of $94\mu\Omega \cdot cm$, and a low temperature coefficient of resistance of 880 ppm/K. However, assembling a mercury ohm with the proper length of 106.300 cm and mass of 14.4521 g was a difficult process. In addition, problems associated with non-uniformities in the glass tube holding the mercury, temperature of the mercury, and end lead effects limited the accuracy of the mercury ohm to about 20 ppm relative to the fundamental cgs ohm.

5.7.2 Manganin Material Alloy

Research on zero Temperature Coefficient of Resistance (TCR) by Weston in the U.S. and Feussner in Germany lead to a material known as manganin in 1890. Manganim is an alloy of approximately 84% copper, 12% manganese, and 4% nickel. Manganin wire was used in the construction of the Reichsanstalt, Rosa, and Thomas wire wound resistors used by the NBS from 1901 until replacement in 1990 by the Quantum Hall Effect standard.

5.7.3 Refchsanstalt Wire Wound Resistor

From 1901 to 1909 the NBS Ohm was maintained as the mean value of 5 to 10 Refchsanstalt wire wound resistors. These resistors were manufactured in Germany and consisted of silk covered manganin wire wound bifilarly on a silk covered and shellac varnished brass cylinder. Each resistor was mounted in a perforated metal can to protect the resistor windings while permitting air to circulate over the resistor windings.

5.7.4 Rosa Wire Wound Resistor

Over time it was discovered that seasonal variations in atmospheric humidity caused small changes to occur in the resistance of Refchsanstalt resistors. The Rosa resistor solved this problem. The material and construction of the Rosa resistor was similar to the Refchsanstalt resistor. However, the Rosa resistor was much smaller allowing it to be enclosed in an oil filled hermetically sealed metal can. A number of Rosa resistors were taken to Europe and compared with the standard mercury ohm maintained in England and also the Germany mercury ohm. The difference in value between the Rosa resistor and the standard mercury ohm was less than 26 ppm. The mean value of ten Rosa one ohm resistors was used as the NBS standard ohm from 1909 to 1930.

5.7.5 Thomas Type I Wire Wound Resistor

The Thomas resistor adopted by the NBS in 1930 provided better stability than the Rosa resistors. The Type I Thomas resistor was constructed of #16 AWG bare manganin wire wound on a 6 cm diameter silk insulated inner brass cylinder. This unit was slipped inside an outer brass cylinder and the entire assembly sealed. The mean value of ten Thomas Type I resistors was used as the NBS standard ohm from 1932 to 1939.

5.7.6 Thomas Type II Wire Wound Resistor

In 1939, slightly larger Thomas Type II resistors, providing better cooling characteristics, replaced the Type I resistors. The mean value of 10 Type II resistors were used as the NBS standard ohm from 1939 to 1969. In September 1969 the 10 Type II resistors were divided into two sub-groups of 5 each to accommodate new measurement equipment designed to work with 5 resistors. In 1971 one of the sub-groups of 5 resistors was over heated and damaged by a malfunction in the controller circuit for the oil bath. The remaining sub-group of 5 resistors was used as the standard from 1972 until they were replaced in 1990 when the International Ohm was redefined based on the quantum Hall effect.

5.8 The United States Standard Volt

The 1893 International Electrical Congress established the International Volt to be $\frac{1000}{1434}$ of the emf produced by the Clark cell at 15^{o} C. In the years that followed experiments showed that the Weston cell was superior in performance to the Clark cell. The 1908 London International Conference on Electrical Units and Standards accepted the Weston Cell as the new reference for the International Volt. The emf of the Weston cell had previously been determined to be 1.0184 volts at 20^{o} C. Later experiments performed using the silver coulometer showed that the emf produced by the Weston cell was 1.0183 volts at 20^{o} C. This new value was accepted in 1910. The Weston cell served as the reference for the International Volt from 1911 to 1948. In 1948 the 9th General Conference on Weights and Measures adopted the Giorgi MKSA system of units. The emf of the Weston cell was determined to be 1.01864 volts at 20^{o} C using the new MKSA system of units.

The mean voltage of seven Clark cells was maintained as the unit of electromotive force in the United States from 1897 to 1906. This mean voltage was assigned the value of 1.4337 volts. The National Bureau of Standards used a combination of Clark and Weston cells from 1906 to 1908. Weston cells were used exclusively by the NBS from 1908 until just recently. The standard volt in the United States is current defined in terms of the quantum based Josephson Effect. Prior to that change, the standard volt was determined by the mean value of 44 Weston cells.

6 1948 General Conference of Weights and Measures (CGPM)

The scope of the Convention du Metre treaty was expanded by the 6th CGPM in 1921 to include other fields of physics beyond the three dimensional MKS (meter, kilogram, second) mechanical system of units. The Giorgi proposal was thoroughly discussed by the 7th CGPM in 1927 as well as by the International Electrical Congress and other international organizations. In 1939 the Consultative Committee for Electricity (CCE), under the direction of the International Committee for Weights and Measures (CIPM), proposed adoption of the Giorgi four dimensional system based on the meter, kilogram, second, and ampere (MKSA). This proposal was approved by the CIPM in 1946. By accepting the Giorgi system, the CIPM eliminated the complex equations defining the electrical units based on the cgs units of length, mass, and time that had been in affect since 1881.

The Giorgi MKSA system was formally accepted in 1948 by 9th General Conference of Weights and Measures (CGPM). The electrical units were specified as follows:

- Ampere (unit of electric current) – The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \cdot (10^{-7})$ MKS unit of force (newton) per meter of length.

- Volt (unit of potential difference and of electromotive force) – The volt is the potential difference between two points of a conducting wire carrying a constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt.

- Ohm (unit of electric resistance) – The ohm is the electric resistance between two points of a conductor when a constant potential difference of 1 volt, applied to these points produces in the conductor a current of 1 ampere, the conductor not being the seat of any electromotive force.

- Coulomb (unit of quantity of electricity) – The coulomb is the quantity of electricity carried in 1 second by a current of 1 ampere.

- Farad (unit of capacitance) – The farad is the capacitance of a capacitor between the plates of which there appears a potential difference of 1 volt when it is charged by a quantity of electricity of 1 coulomb.

- Henry (unit of electric inductance) – The henry is the inductance of a closed circuit in which an electromotive force of 1 volt is produced when the electric current in the circuit varies uniformly at the rate of 1 ampere per second.

- Weber (unit of magnetic flux) – The weber is the magnetic flux which, linking a circuit of one turn, would produce in it an electromotive force of 1 volt if it were reduced to zero at a uniform rate in 1 second.

In 1954 the 10th CGPM approved the introduction of the ampere, the kelvin, and the candela as base units, respectively, for electric current, thermodynamic temperature, and luminous intensity. This decision brought the number of base units to six, length, mass, and time being the first three.

In 1960 the 11th CGPM renamed the MKSA system to the International System of Units (SI).

In 1971 at the 14 CGPM the mole was added as the seventh and last base unit of the SI system.

7 2014 General Conference of Weights and Measures (CGPM)

Historically the base units of the SI system have been defined by manufactured physical objects designated as the official international standards. For example, the International Prototype Meter was defined at the 1st CGPM in 1889 as the distance between two lines on a standard bar composed of an alloy of 90% platinum and 10% iridium measured at the melting point of ice. Similarly, the International Prototype Kilogram was defined as a right-circular cylinder, made of a platinum alloy consisting of 90% platinum and 10% iridium, 39.17 millimeters in height and diameter. The International Standard Second was defined as $\frac{1}{86{,}400}$ of a Solar Day. The problem with these standards is that they are not very precise relative to current state of the art measurement technology. In addition they are not stable. Because they are physical objects they are subject to physical change, that is they age ever so slightly over time. Consequently they are not stable or precise enough to serve as standard units for the level of scientific research and engineering work being performed today.

The approach to resolve this problem has been to redefine the SI base units in terms of physical constants occurring in nature. This trend began in 1967 in redefining the second as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the caesium-133 atom ground state. The definition of the meter was changed in a similar manner in 1983 to be the length of the path travelled by light in a vacuum during a time interval of $\frac{1}{299{,}792{,}458}$ of a second.

The 25th CGPM has extend this approach to include the remaining SI base units, specifically redefining the kilogram, ampere, kelvin, mole and candela in terms of physical constants occurring in nature. The physical constants used in defining the SI base units are:

- The unperturbed ground state hyperfine splitting frequency of the caesium 133 atom which is exactly 9,192,631, 770 hertz,

- The speed of light in vacuum (the symbol c) which is exactly 299,792,458 meters per second,

- The Planck constant (the symbol h) which is exactly $6.62606957 \cdot (10^{-34})$ joule second,

- The elementary charge (the symbol e, the charge of an electron or proton) which is exactly $1.602176565 \cdot (10^{-19})$ coulomb,

- The Boltzmann constant (the symbol k) which is exactly $1.3806488 \cdot (10^{-23})$ joule per kelvin,
- The Avogadro constant (symbol N_A) which is exactly $6.02214129 \cdot (10^{23})$ 20^{o} C reciprocal mole,
- The luminous efficacy (symbol K_{cd}) of monochromatic radiation of frequency $540 \cdot (10^{12})$ hertz which is exactly 683 lumen per watt.

The definitions provided by the 25th CGPM for the seven SI base units are described below:

7.1 Second (SI unit of time)

The definition for the second remains the same as defined in 1967. That is, one second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the caesium-133 atom ground state. The rigorous wording approved by the 25th CGPM is:

The second, symbol s, is the SI unit of time; its magnitude is set by fixing the numerical value of the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom to exactly 9,192,631,770 when it is expressed in the SI units (s^{-1}), which for periodic phenomena is equal to Hz.

The number of periods of radiation was selected so that this definition of 1 second corresponds to the duration of 1 second originally defined in 1874 and revised in 1960 to achieve better accuracy.

7.2 Meter (SI unit of length)

The definition for the meter remains the same as defined in 1983. That is, one meter is the length of the path travelled by light in vacuum during the time interval of $\frac{1}{299,792,458}$ of a second. The more rigorous wording approved by the 25th CGPM is:

The meter, symbol m, is the SI unit of length; its magnitude is set by fixing the numerical value of the speed of light in a vacuum to be exactly 299,792,458 when it is expressed in the SI unit for speed $\boldsymbol{m \cdot (s^{-1})}$.

It follows that the speed of light (c) in a vacuum is exactly 299,792,458 meters per second. That is c = 299,792,458 m/s.

The value $\frac{1}{299,792,458}$ was selected so that this definition of the meter corresponds to the length of the International one meter prototype accepted at the 1st CGPM in 1889.

Note that defining the meter in terms of the speed of light results in the definition for length depending on the definition of time.

7.3 Kilogram (SI unit of mass)

The definition of the kilogram has undergone a fundamental change. Previously the kilogram was defined as the mass of the international prototype of the kilogram maintained at the International Bureau of Weights and Measures (BIPM) in Sevres, France.

The new definition of the kilogram is defined in terms of the Planck constant. The numerical value of the Planck constant is chosen so that the new definition for the kilogram is equal to the mass of the international prototype kilogram within a few parts in 10^8. A few parts in 10^8 is the uncertainty of the combined best estimates for the value of the Planck constant at the time of the 25th CGPM.

The new definition for the kilogram is:

The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be exactly $\boldsymbol{6.62606957 \cdot (10^{-34})}$ when it is expressed in the SI unit for action $\boldsymbol{J \cdot s = kg \cdot m^2 \cdot (s^{-1})}$.

The new definition for the kilogram (mass) is thus dependant on the definition of both time and length (seconds and meters).

7.4 Ampere (SI unit of electrical current)

The ampere was previously defined, in October 1948 by the 9th CGPM, as that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \cdot (10^{-7})$ newton per meter of length. This definition has been replaced with the following.

The ampere, symbol A, is the SI unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be exactly $\boldsymbol{1.602176565 \cdot (10^{-19})}$ when it is expressed in the SI unit for electric charge C = As.

From this new definition:

- The value of the elementary charge (the charge of an electron or proton) is fixed at exactly $e = 1.602176565 \cdot (10^{-19})C = 1.602176565 \cdot (10^{-19})As$ where C is coulomb, A is ampere, and s is seconds.

- The ampere is the electric current corresponding to the flow of

$$\frac{1}{1.602176565 \cdot (10^{-19})}$$

elementary charges per second.

Note that the new definition of the ampere (electrical current) is dependant on the definition of time (seconds).

The previous definition of the ampere based on the force between current carrying conductors fixed the value of the magnetic permeability in a vacuum at $\mu_0 = 4\pi(10^{-7})$ henry per meter.

The new definition for the ampere fixes the value of e instead of μ_0. Consequently μ_0 is no longer an exact quantity but must be determined experimentally. The experimental value for μ_0 is the same as its previous value, however, this value now has an uncertainty associated with it instead of it being an absolute value. The value for μ_0, at the time the new definition for the ampere was adopted, is $\mu_0 = 4\pi \cdot (10^{-7})$ henry per meter with a relative standard uncertainty of less than $1 \cdot (10^{-9})$.

The electric permittivity in a vacuum is

$$\varepsilon_0 = \frac{1}{\mu_0 \cdot c^2} = 8.854 \cdot (10^{12})$$

farad per meter

and thus has the same uncertainty as μ_0 since the speed of light (c) is exactly know. Similarly the characteristic impedance of a vacuum is

$$Z_0 = \mu_0 \cdot c$$

and also has the same uncertainty as μ_0. The product

$$\varepsilon_0 \cdot \mu_0 = \frac{1}{c^2}$$

remains exactly known as well as the quotient

$$\frac{Z_0}{\mu_0} = c$$

since the value of c is exactly known.

The 1948 theoretical definition of the ampere was difficult for national laboratories to implement in practice. By 1987, the standard volt defined by the quantum based Josephson Junction and standard ohm defined by the quantum-Hall effect, were being used nearly universally by national labs to define the ampere through Ohm's law.

7.5 Kelvin (SI unit of thermodynamic temperature)

The kelvin was previously based on the triple point of water. The new definition of the kelvin is based on the Boltzmann constant. Specifically:

The kelvin, symbol K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be exactly $\mathbf{1.3806488 \cdot (10^{-23})}$ when it is expressed in the SI unit for energy per thermodynamic temperature $\boldsymbol{J \cdot (K^{-1}) = kg \cdot m^2 \cdot (s^{-2})(K^{-1})}$.

The effect of this definition is that the kelvin is equal to the change of thermodynamic temperature that results in a change of thermal energy kT by $1.3806488 \cdot (10^{-23})$ joules.

The previous definition of the kelvin was based on an exact value assigned to the triple point of water, specifically $T_{TPW} = 273.16K$. Because the new definition of the kelvin fixes the value of Boltzmann constant k instead of the triple point of water T_{TPW}, T_{TPW} must now be determined experimentally. The Boltzmann constant was selected so that T_{TPW} is equal to 273.16 K (its previous value), with a relative standard uncertainty of less than $1 \cdot (10^{-6})$ based on measurements of k made prior to the redefinition.

7.6 Mole (SI unit of amount of substance)

The definition of the mole has been changed to the following.

The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be exactly $\mathbf{6.02214129 \cdot (10^{23})}$ when it is expressed in the SI unit $\boldsymbol{(mol^{-1})}$.

The effect of this definition is that the mole is the amount of substance of a system that contains $6.02214129 \cdot (10^{23})$ specified elementary entities.

7.7 Candela (SI unit of luminous intensity)

The definition of the candela has been changed to the following.

The candela, symbol cd, is the unit of luminous intensity in a given direction; its magnitude is set by fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency $\mathbf{540 \cdot (10^{12})}$ Hz to be exactly 683 when it is expressed in the SI unit $\boldsymbol{(kg^{-1})(m^{-2})(s^{3})(cd)(sr) = lm \cdot (W^{-1}) = cd \cdot sr \cdot (W^{-1})}$.

The effect of this definition is that the candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot (10^{12})$ Hz and that has a radiant intensity in that direction of (1/683) W/sr, where sr (steradian) is squared radians.

7.8 Derived Units

The definitions of all of the SI derived units (newton, joule, watt, volt, ohm, etc.) remain the same since they are defined entirely in terms of the SI base units.

8 Summary of Electrical Units as Currently Defined.

8.1 Ampere

In theoretical terms, the ampere is the electric current corresponding to the flow of $1/1.602176565 \cdot (10^{-19})$ elementary charges per second.

Operationally the ampere is defined through Ohm's law using the value of the volt and the ohm defined respectively by the quantum based Josephson effect and the quantum-Hall effect.

8.2 Volt

In theoretical terms, the volt is defined as the potential difference between two points of a conducting wire carrying a constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt.

The volt is currently defined operationally by the quantum based Josephson Effect which provides an accuracy of 1 part per billion. The Josephson Effect provides an exact reproducible relationship between frequency and voltage known as the Josephson constant. The Josephson constant $K_{(J-90)} = 483{,}597.9\ GHz/V$. Since frequency is precisely defined by the caesium standard, the Josephson Effect can be used to provide a practical and precise definition of a volt.

8.3 Ohm

In theoretical terms, the ohm is defined as the electric resistance between two points of a conductor when a constant potential difference of 1 volt, applied to these points produces in the conductor a current of 1 ampere.

The ohm is currently defined operationally by the quantum-Hall effect which provides an accuracy of nearly 1 part per billion. The quantum-Hall effect von Klitzing constant is

$$R_{(K-90)} = 25{,}812.807\ \Omega$$

8.4 Coulomb

The coulomb is the quantity of electricity carried in 1 second by a current of 1 ampere.

$$One\ coulomb = \frac{1}{1.602176565 \cdot (10^{-19})}$$

elementary charges

8.5 Farad

The farad is the capacitance of a capacitor between the plates of which there appears a potential difference of 1 volt when it is charged by a quantity of electricity of 1 coulomb.

8.6 Henry

The henry is the inductance of a closed circuit in which an electromotive force of 1 volt is produced when the electric current in the circuit varies uniformly at the rate of 1 ampere per second.

8.7 Weber

The weber is the magnetic flux which, linking a circuit of one turn, would produce in it an electromotive force of 1 volt if it were reduced to zero at a uniform rate in 1 second.

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