Mechanical Waves

www.skywave-radio.org

1 Mechanical Waves

Mechanical waves require some type of physical medium for the waves to travel through, for example air, water, a plasma, and numerous other physical substances. In contrast, a medium is not required for the propagation of electromagnetic waves. For instance, electromagnetic waves travel unhindered through the vacuum of outer space. Electromagnetic waves include X-rays, Extreme Ultra Violate (EUV) radiation, visible light, infrared, radio waves, etc.

Mechanical waves transport energy in the form of disturbances which propagate through a medium without any corresponding net motion of the medium itself. Water molecules move upward and then back down to their original positions as a wave propagates through the water. A disturbance will cause a wave to propagate though a material provided that the material has both

- Inertia, and
- Elasticity.

From Newton's first law of motion, inertia is the property of an object, including a particle, which causes it to remain at rest or in uniform motion in a straight line until acted upon by an external force.

Elasticity is a restoring force inherent in a material which causes it to return to its original shape after being deformed by some external force. Note that if the external force is great enough it will exceed the material's elasticity causing the material to be permanently deformed.

The restoring force of elasticity cause particles within a material to oscillate about their central undisturbed position after being displaced by a transient external force. The oscillations slowly dampen out due to internal friction as the particles return to their normal state. However, as they oscillate they excite adjacent particles causing a mechanical wave to propagate outward through the material from the point of disruption. It is important to note that the particles composing a material simply oscillate up and down, or back and forth, about their undisturbed neutral location as the wave passes by. The particles do not travel along with the wave.

Energy can be transmitted a considerable distance by wave motion. The energy in a wave is the kinetic and potential energy of the material particles that is passed on from one particle to the next as the wave flows through the material.

2 Transverse and Longitudinal Waves

Mechanical waves can be either:

- Transverse, or
- Longitudinal.

The particles in a transverse wave oscillation perpendicular to the direction of wave travel as shown in Figure 1. For example, if a horizontal rope under tension is moved quickly up and down at one end, as in Figure 2, a transverse wave will travel down the length of the rope. The wave disturbance moves down the rope but the particles making up the rope simply oscillate up and down as the wave passes.

A longitudinal wave is characterized by particles oscillating back and forth in the direction of wave travel as shown in Figure 1. Consequently zones of particle compression (particles become closer together) and rarefication occur in longitudinal waves, as illustrated in Figure 3 for a sound wave. In the case of a sound wave it is air particles that oscillate back and forth as the wave passes.

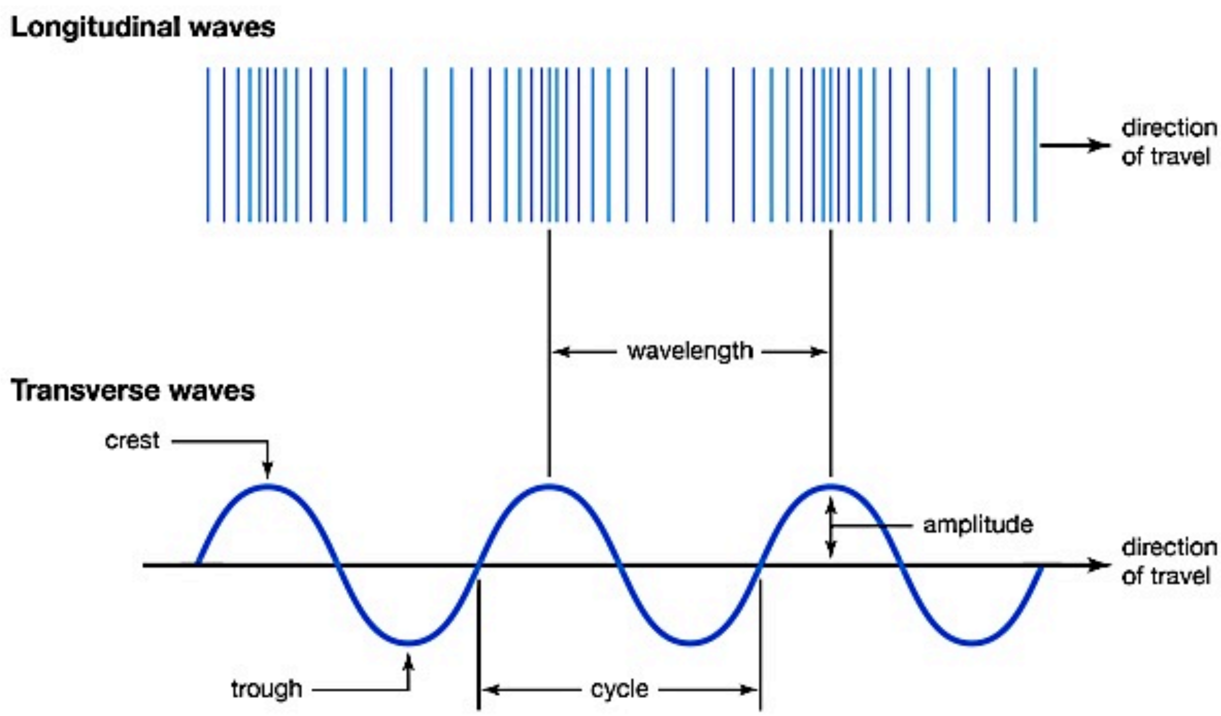


Figure 1 Transverse vs Longitudinal Wave (source Encyclopedia Britannica)

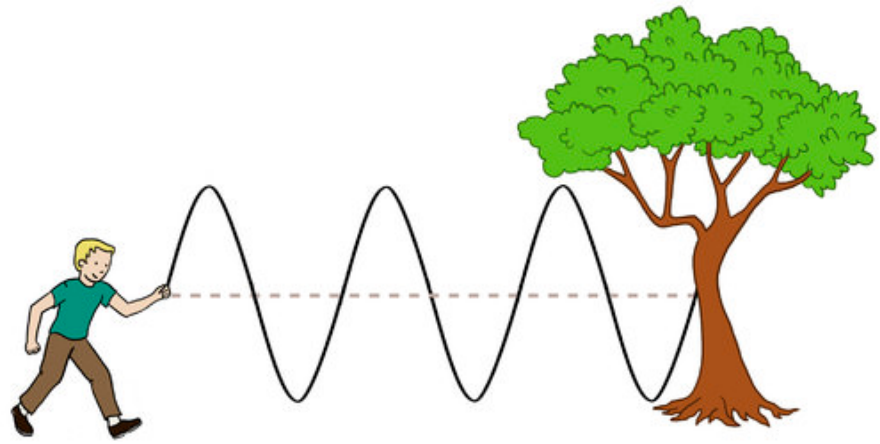


Figure 2 Using a rope to create a transverse wave (source: ck12.org)

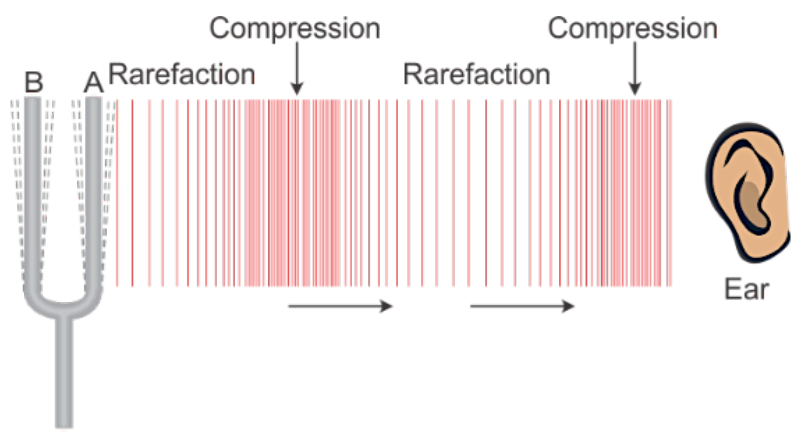


Figure 3 Longitudinal sound wave (source: tuttee.co)

A wave can be a single pulse or a train of waves. For example, a transverse pulse will be produced if the end of a taut rope is moved quickly upward and down once, as in Figure 4. Each particle comprising the rope remains at rest until the pulse reaches it. It then moves upward and back down in the short period required for the wave to pass. After the wave has passed, the particle returns to its original state of rest. A train of waves will be produced if the end of the rope is continuously moved up and down as in Figure 2. If the motion in moving the rope is periodic, then a periodic train of waves will be produced. In this case the particles of the rope oscillate up and down in simple harmonic motion.

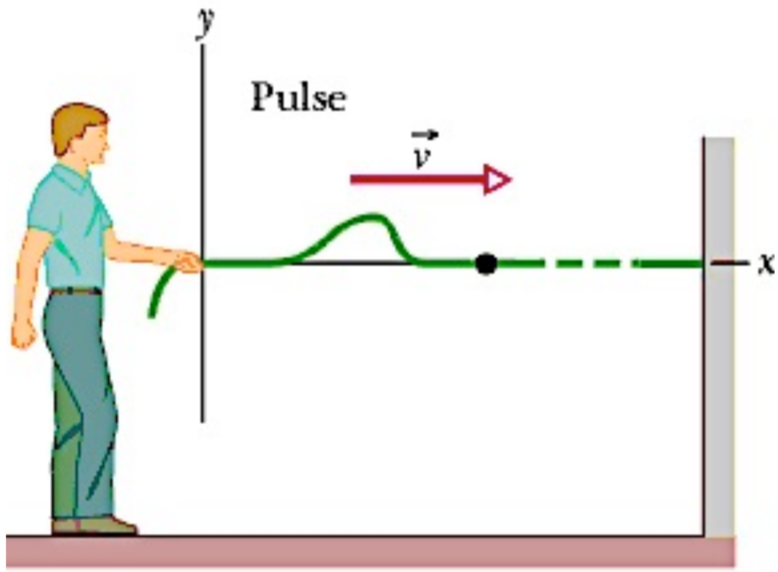


Figure 4 Pulse wave (source: www.dzre.com)

3 Wavefronts

In discussing waves, both mechanical and electromagnetic, the concept of a wavefront is very important. A wavefront is defined as a surface perpendicular to the direction of wave motion in which the phase angle of the wave is the same at all points on the surface. A wave consists of an infinite number of wavefronts, one for each phase angle and fraction there of. Thus in discussing a wave we generally talk about a specific wavefront. For example, Figure 5 illustrates a number of wavefronts for a transverse sinusoidal wave. Notice that pink wavefronts are located along the wave such that the wave phase angle is 90^o at each pink wavefront. In addition, the pink wavefronts travel along with the wave so that at any point in time the phase angle of the wave passing through a pink surface is always 90^o. Similarly, the greenish wavefronts travel with the wave so that at any point in time the phase angle of the wave passing through a greenish surface is always 270^o.

Wavefronts can have many shapes. A plane wave travels in a single direction and has a wavefront which is a flat surface perpendicular to the direction of wave motion. For spherical waves the disturbance propagates radially outward in all direction from a single point source. In this case wavefronts are concentric spheres (or circles in two dimensions) centered on the point source. In Figure 6 a plane wave is approaching a barrier from the left. As it passes through a small “pin hole” in the barrier, the wave is transformed into a spherical wave. The spherical wave propagates out from the pin hole on the right side of the barrier.

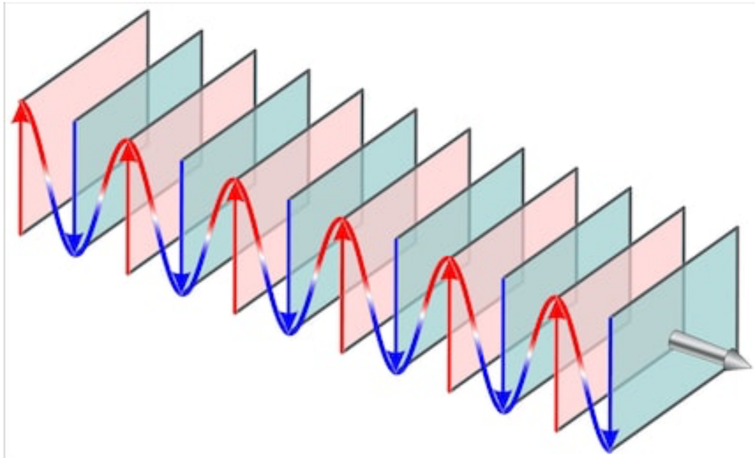


Figure 5 Wavefronts (source: shutterstock.com)

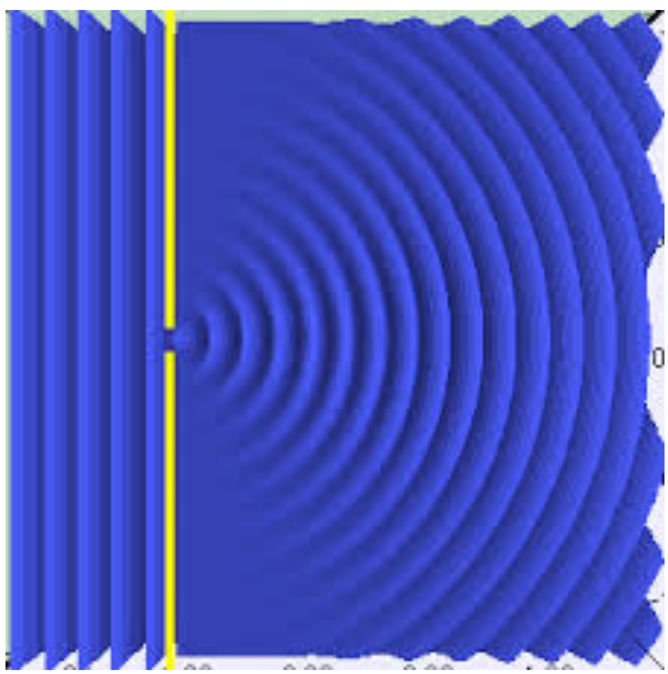


Figure 6 Plane and Spherical Wavefronts (source: LibGuides)

4 Transverse Waves

4.1 Transverse Wave Equations

Figure 7 shows a transverse sinusoidal wave “frozen” in time. The equation for this wave is

$$y = y_m \sin\left[2\pi \frac{x}{\lambda}\right]$$

where

x = distance from the origin 0 in the horizontal direction

y = the displacement of the wave from the x axis

y_m = the maximum displacement, or **wave amplitude**

$\left[2\pi \frac{x}{\lambda}\right]$ = phase angle of the wave in radians

λ = the wavelength of the wave

Wavelength $\boldsymbol{\lambda}$: Wavelength is the distance in the x direction from any arbitrary point on the wave to the next corresponding point at the same phase angle. For convenience in Figure 7 the wavelength is measured from a wave crest (phase angle = 90^o) to the next wave crest.

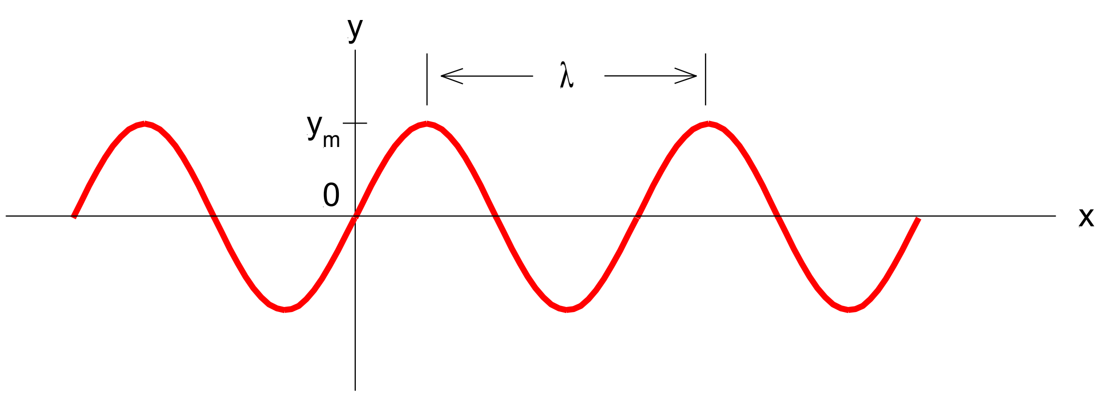


Figure 7 A periodic wave frozen in time.

At the origin (x = 0) the wave displacement is

$$y = y_m \sin\left[2\pi \frac{x}{\lambda}\right] = y_m \sin\left[2\pi \frac{0}{\lambda}\right] = y_m \sin 0 = 0$$

as shown in Figure 7. At a phase angle of 90^o, that is $\left[\frac{1}{4}\lambda\right]$ radians, the wave displacement is

$$y = y_m \sin\left[2\pi \frac{x}{\lambda}\right] = y_m \sin\left[2\pi \frac{1\lambda}{4\lambda}\right] = y_m \sin\frac{1}{2}\pi = y_m \cdot 1 = y_m$$

again as shown in Figure 7.

Phase Velocity: In Figure 8 the wave moves to the right at a velocity of v. To avoid deforming the wave, every point on the wave must move to the right at the same velocity. This velocity is known as the Phase Velocity.

$$\boldsymbol{phase\ velocity\ \ v} = \frac{distance\ traveled\ (x)}{second}$$

Group Velocity: In many situations a disturbance creates a number of different waves instead of a single wave. In this case the energy associated with the disturbance is transported at a velocity that is different from the phase velocity of any individual wave. Instead energy propagates through the medium at a velocity known as the group velocity. In the world of electromagnetic waves, a voice modulated radio signal consists of a carrier wave, an upper sideband wave, and a lower sideband wave each with its own phase velocity. However, the energy associated with the radio signal travels at a group velocity that is different from any of the three phase velocities.

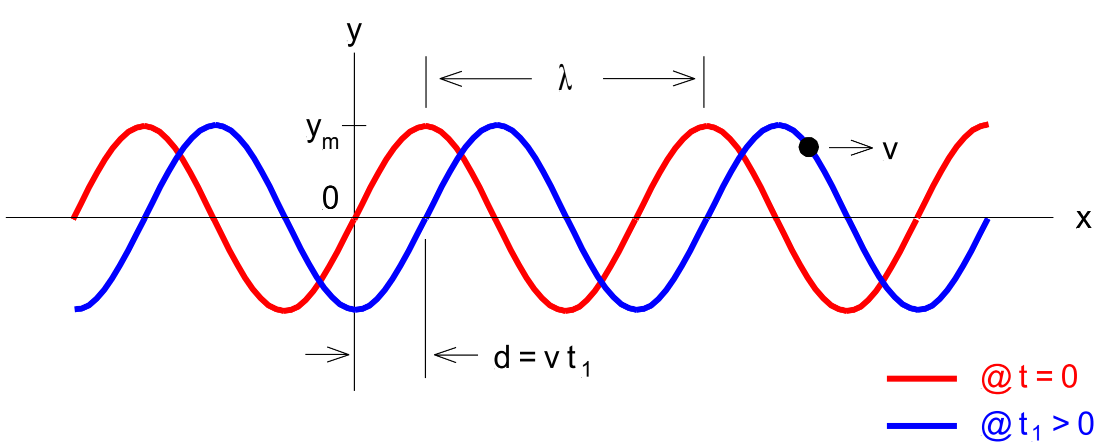


Figure 8 Traveling Wave

Traveling Wave: The equation for a sinusoidal wave moving in the x direction through some medium is

$$y = y_m \sin\frac{2\pi}{\lambda}[x - vt]$$

when the wave is propagating to the right at a phase velocity v. For a wave propagating to the left, the equation becomes

$$y = y_m \sin\frac{2\pi}{\lambda}[x + vt]$$

That is, the term vt is added to x instead of being subtracted from it.

In Figure 8 the red wave represents the wave at time t = 0 (that is at t_0). The blue wave represents the position of the wave at some later time $t_1 > 0$. As shown in the figure, the wave has moved to the right a distance of $d = vt_1$ in the time interval $t_1 - t_0$.

Phase Angle $\boldsymbol{\phi}$: The above equations assume that the displacement y is zero at the position x = 0 at the time t = 0. This of course does not need to be the case. The general equation for a sinusoidal wave moving to the right is

$$y = y_m \sin\frac{2\pi}{\lambda}[x - vt - \phi]$$

where ϕ is the phase angle. If $\phi = -90° = -\frac{1}{4}\lambda$ radians, then the displacement of y at x = 0 and t = 0 is

$$y = y_m \sin\frac{2\pi}{\lambda}[x - vt - \phi] = y_m \sin\frac{2\pi}{\lambda}\left[0 - 0 - \left(\frac{-1}{4}\lambda\right)\right] = y_m \sin\frac{1}{2}\pi = y_m$$

A number of wave properties are useful.

Wave Period: The period T is the time required for the wave to travel a distance of one wavelength λ at a velocity of v. Thus

$$\boldsymbol{period\ T} = \frac{\lambda}{v}\ seconds$$

Wave Frequency: Wave frequency is the number of wavelengths, or cycles, traversed in one second. Thus

$$\boldsymbol{frequency\ f} = \frac{1}{T}\ hertz$$

Substituting in the expression for period T and rearranging terms gives the equation for wavelength with respect to frequency and phase velocity. Thus:

$$\boldsymbol{wavelength\ \lambda} = \frac{v}{f}$$

Angular Frequency: It is often convenient to express frequency in terms of angular frequency (ω) with units of radians per second. In this case

$$\boldsymbol{angular\ frequency\ \omega} = \frac{2\pi}{T} = 2\pi f$$

Finally, a wave number is often specified for a sinusoidal wave in which

$$\boldsymbol{wave\ number\ \ k} = \frac{2\pi}{\lambda}$$

Using these definitions the equation for a traveling sinusoidal wave moving to the right is

$$y = y_m \sin[kx - \omega t - \phi]$$

4.2 Superposition of Waves

In many cases two or more waves can travel through the same medium at the same time completely independent of one an other. For example, we can discern the notes of a particular instrument in an orchestra even through multiple sound waves from many instruments are traveling through the air at the same time. The same is true of light. At any given time the space around us is filled with electromagnetic waves from an almost infinite number of energy sources including the Sun, heat radiating from hot surfaces, reflected light from visible objects, radio waves, etc. The entire purpose of a radio receiver is to select one specific radio signal to listen to from the thousands of radio waves arriving at the radio antenna.

The fact that waves traveling through a medium act independently of each other leads to the concept of wave superposition. Specifically, the total displacement of a particle at any given time is simply the sum of the displacements produced by each individual wave traveling through the medium.

However, the principle of superposition applies only if the mathematical relationship between a disrupting force and the elastic restoring force of the material is a simple linear equation. That is, the restoring force must be proportional to the disturbance. As an example, the restoring force in a mechanical spring is proportional to the force F disrupting the spring in accordance with Hooks Law

$$F = -kx$$

In this equation k is the spring constant and $-kx$ is the restoring force. Consequently, if two different forces F_1 and F_2 are exciting a spring at the same time, two waves will travel through the spring. In equation form

$$Wave\ 1\text{:}\ F_1 = -kx_1 \qquad and \qquad Wave\ 2\text{:}\ F_2 = -kx_2$$

Using superposition, the total displacement of the spring is simply

$$x_{Total} = x_1 + x_2 = -\frac{[F_1 + F_2]}{k}$$

An other example is the human ear. The ear drum in the human ear responds linearly to sound waves reaching it permitting our brain to process various sounds, for example, picking out the notes from a particular instrument in an orchestra. However, our ear drums will be driven out of their linear range of operation if two very loud notes are played. When this happens the superposition principle no longer applies. Instead our ears erroneously manufacture additional sounds that were never played. These manufactured sounds combine with the two loud sounds producing a very unpleasant noise. This phenomena is known as intermodulation distortion.

Yet an other example is that of an explosion. A violent explosion creates a shock wave. While shock waves are elastic longitudinal waves traveling through air, they behave differently from ordinary sound waves. The equation describing propagation of a shock is a non-linear quadratic equation. Consequently, superposition does not apply to shock waves.

4.3 Wave Interference

Wave interference is the result of superimposing two or more waves.

Assume that two sine waves (Wave 1 and Wave 2) with the same frequency and phase velocity are propagating through a medium at the same time. The motion of particles in the medium is determined by superimposing (adding together) the two waves. When this is done the result is a third sine wave of the same frequency and phase velocity as Wave 1 and Wave 2, but with a different amplitude and phase angle. The resulting third wave describes the actual motion of the particles within the medium.

If Wave 1 (green wave) and Wave 2 (blue wave) in Figure 9 are in phase and have the same amplitude y_m , the resulting red wave will also be in phase but have twice the amplitude ($2y_m$). Figure 9 is referred to as **constructive interference**. However, if Wave 1 and Wave 2 in Figure 10 are 180^o degrees out of phase, the amplitudes of the two waves will completely cancel producing no resulting wave. Figure 10 is an example of **destructive interference**.

The amplitude of the resulting red wave will be between these two extremes $(0 \leq y \leq 2y_m)$ if

$$amplitude\ Wave\ 1\ \neq amplitude\ Wave\ 2$$

and the phase differences θ between Wave 1 and Wave 2 is

$$0 \leq \theta \leq 180°$$

The phase angle ϕ of the resulting wave will also be different from that of both Wave 1 and Wave 2 as shown in Figure 11.

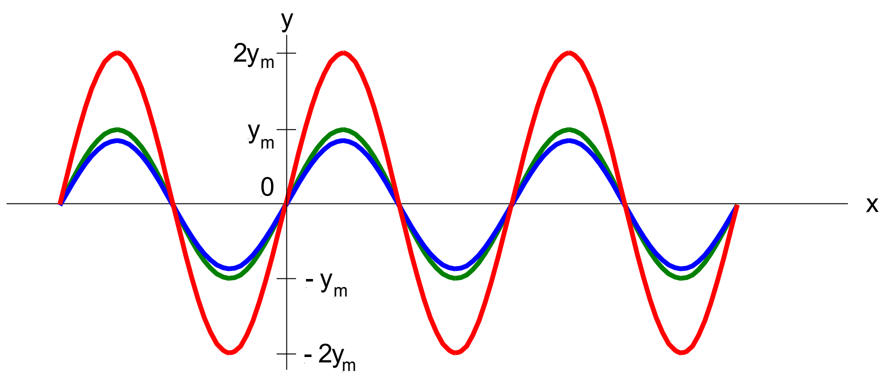


Figure 9 Two waves (blue & green) in phase with same amplitude, frequency, and velocity

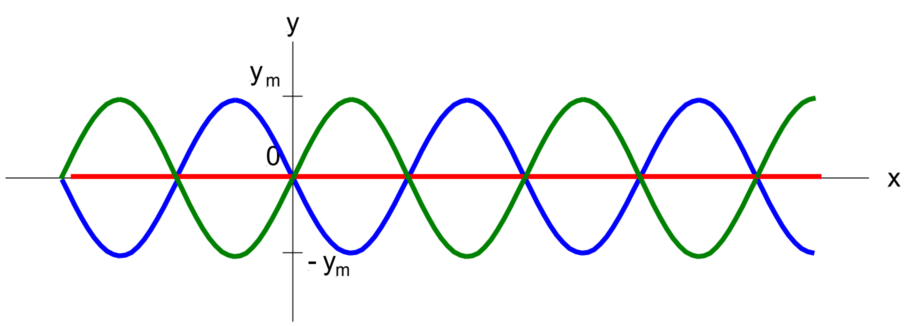


Figure 10 Two waves (blue & green) same amplitude, frequency, and velocity, 180° out of phase

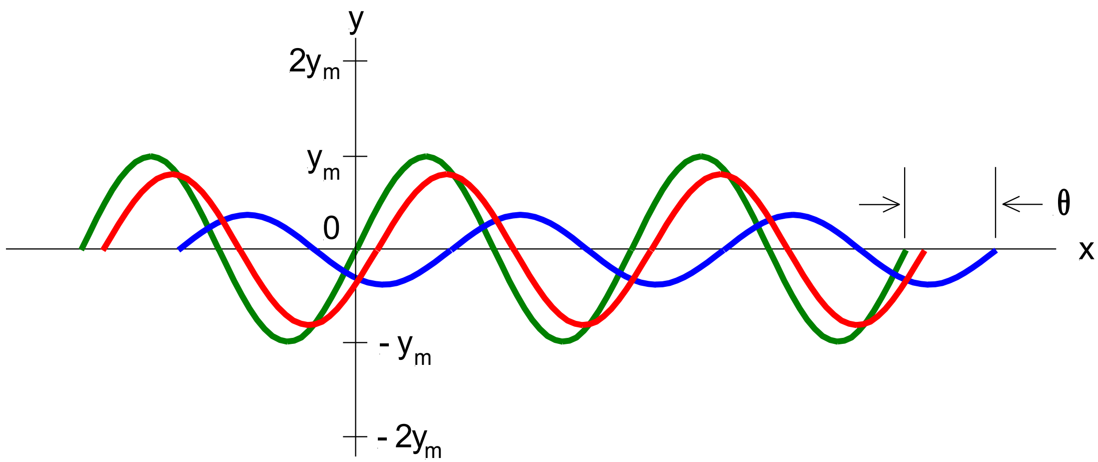


Figure 11 Two waves (blue & green) same freq., and velocity different amplitudes and phase

In general, the superposition of any number of sinusoidal waves of the same frequency and phase velocity, but different amplitudes and phase angles, will produce a resulting sine waves which also has the same frequency and phase velocity.

4.3.1 Phase Difference Due to Path Length

A phase difference can develop between Wave 1 and Wave 2 if they begin as identical waves at their source but follow different paths to the destination. If the path difference is an integer number of wavelengths

$$0, \qquad \lambda, \qquad 2\lambda, \qquad 3\lambda, \qquad etc$$

then the two waves will add constructively at the destination, that is the resulting amplitude at the destination will be twice that of the original waves. However if the path difference is an odd number of half wavelengths

$$\frac{1}{2}\lambda, \qquad \frac{3}{2}\lambda\,, \qquad \frac{5}{2}\lambda, \qquad etc$$

then at the destination the two waves will add destructively, completely canceling each other out resulting in no signal at the destination.

4.3.2 Complex Waves

Superimposing two or more sine waves traveling at the same speed (for example the speed of sound through air), but having different frequencies, produces a resulting wave which is no longer sinusoidal. Instead the resulting wave can be quite complex as shown in Figure 12. In this figure three waves of different frequencies (top trace) are superimposed to produce a complex resultant wave (bottom trace). The motion of medium particles is no longer simply harmonic motion when the resulting wave is complex.

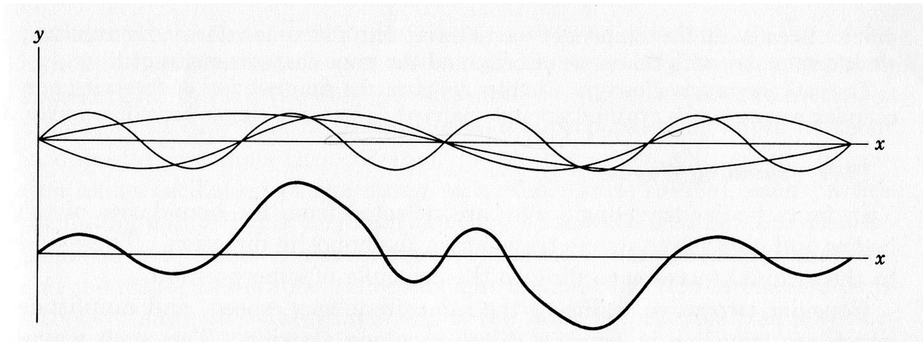


Figure 12 Superposition of 3 waves of different frequencies. (source: Resnick & Halliday)

4.3.3 Fourier Transform

French mathematician J. Fourier (1768 – 1830) showed that any complex periodic waveform can be produced, or represented, by the superposition of harmonically related sine waves, all traveling at the same phase velocity, but each having its own amplitude. In equation form the amplitude y(t) of any periodic wave form is equal to

$$y(t) = A_0 + A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t \cdots + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t \cdots$$

In Figure 13a a sawtooth waveform (dotted line) is produced by the superposition of six sinusoidal wave forms with frequencies of $\omega \ to\ 6\omega$, Figure 13b.

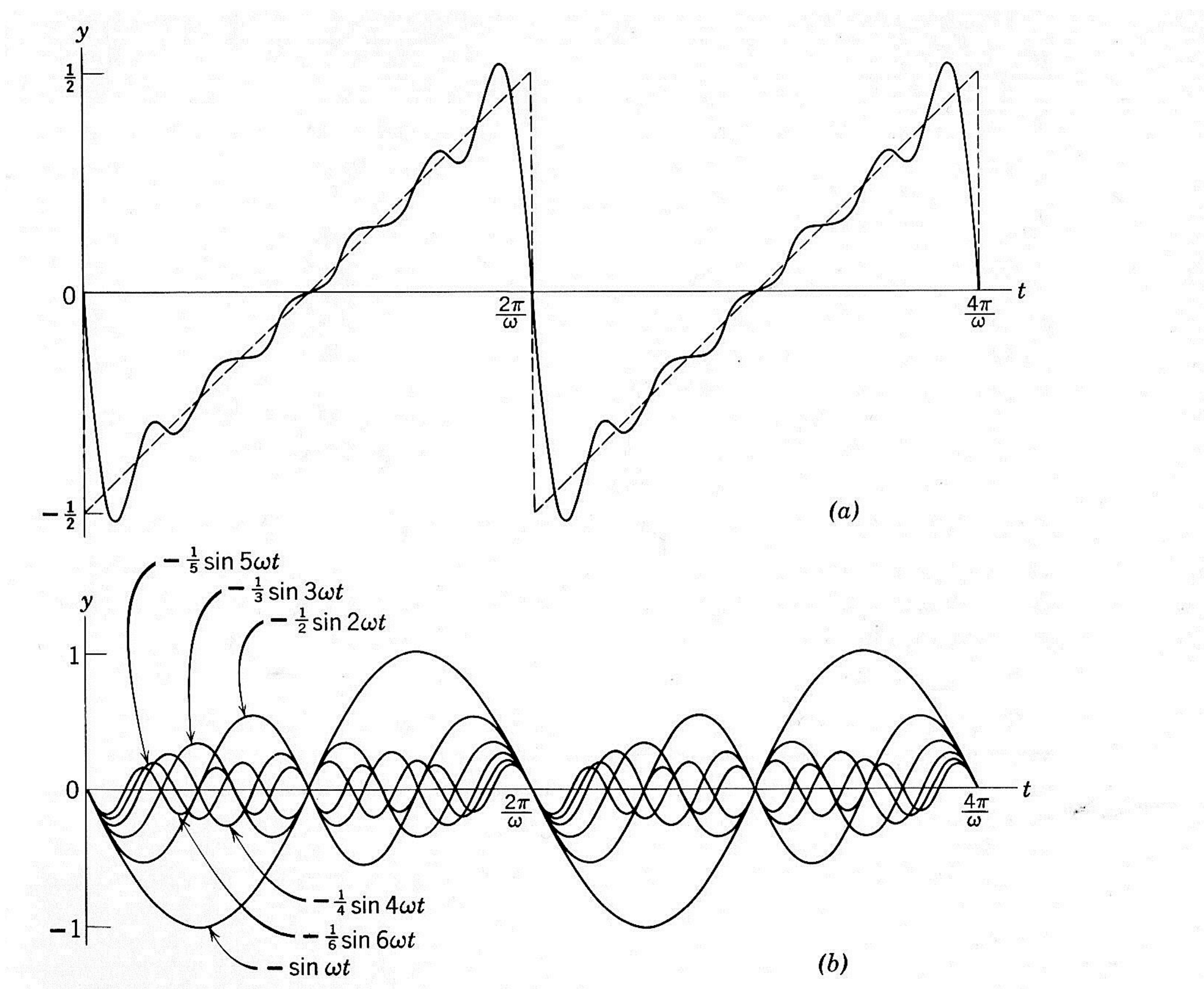


Figure 13 Sawtooth wave from by superposition of 6 sine waves (source: Resnick & Halliday)

4.4 Wave Reflection

A wave is reflected when it reaches the boundary (the edge) of the medium through which it is traveling. It is also reflected if it encounters a discontinuity in the medium, for example a change in the medium’s density.

In Figure 14 a pulse is traveling to the left along a stretched rope which is attached at its far end to an immovable wall. The pulse exerts an upward force on the wall as it reaches the left end of the rope. However the wall is rigid and can not move. According to Newton’s third law of mechanics, the wall exerts an equal and opposite force on the rope. This force generates a downward pulse of the same magnitude as the original incoming upward pulse but traveling to the right back down the rope. The incoming pulse has been reflected by the immovable wall.

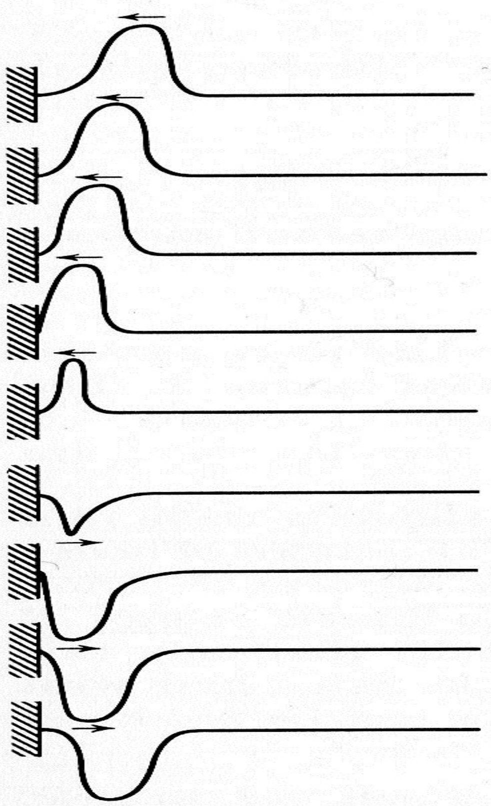


Figure 14 Wave reflection from a fixed boundary (source: Resnick & Halliday)

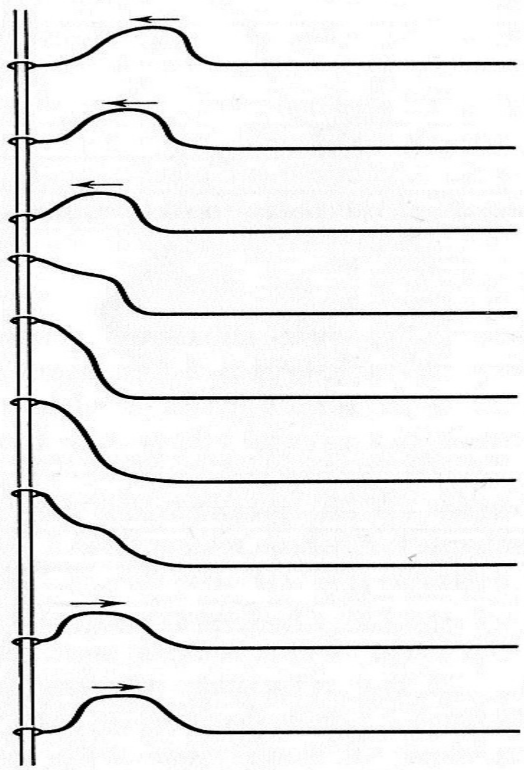


Figure 15 Wave reflection when far end of string is free to move (source: Resnick & Halliday)

The same thing occurs when an incident sinusoidal wave traveling down the rope to the left encounters the wall. The reactive force of the wall generates a reflected wave of the same frequency but opposite in amplitude that travels to the right back down the rope. The displacement of a rope particle at any point along the rope is the sum of the displacement caused by the incident wave and that caused by the reflected wave traveling in the opposite direct. Since the left end of the rope is attached to the wall and can not move either up or down, the sum of the incident and reflected wave at that point must be zero. That is, at the wall the incident and reflected waves must interfere destructively, cancelling each other out. Consequently, for a rope that is fixed at one end, the reflected wave must be 180^{o} out of phase with the incident wave.

The situation is different when the far end of the rope is allowed to move up and down. In Figure 15 the rope is attached to a ring that can move freely (without friction) up and down a vertical rod.

The incident pulse in Figure 15 traveling to the left has a positive (upward) amplitude of y_m. The approaching pulse exerts a force on the particles composing the free end to the rope. The force accelerates the free end of the rope, and its attached ring, upward. Momentum imparted to the rope carries it past the y_m mark on the rod. The rope reaches the $2y_m$ mark just as the maximum amplitude of the incident pulse, y_m, encounters the vertical rod. From that point on the amplitude of the incident pulse decreases to zero causing the free end of the rope to travel back down the rod. The free end of the rope is at the y_m mark on the rod when the amplitude of the incident pulse reaches zero (the incident pulse has passed). Momentum causes the free end of the rope to continue down to the bottom of the rod creating a reflected pulse traveling to the right with an amplitude of y_m as illustrated in Figure 15.

The displacement of the particles in the free end of the rope is the sum of the displacement caused by the incident and reflected pulses. Since the maximum displacement is $2y_m$, the incident and reflected pulse interfere constructively at the rod, that is the incident and reflected pulses are in phase. Consequently, at the free end of the rope the incident pulse is reflected without phase change.

The same phenomena occurs for other types of waves including sine waves. The incident wave is reflected at the free end of the rope without phase change.

4.4.1 Partial Wave Reflection

The reflection scenario just described assumed that incident wave was completely reflected at the far end of the rope. In general this is not the case. Usually a wave will be only partially reflected at a medium boundary or discontinuity with the remainder of the wave traveling on ward past the boundary or discontinuity.

For example, suppose the far end of the rope described above is attached to a second rope with a different density. That is, the second rope is thicker or thinner than the first rope. At the boundary joining the two ropes, the incident wave traveling to the left will be partially reflected and partially transmitted on into the second rope. The amplitude of the reflected wave will be less than that of the incident wave since some of the incident wave energy is carried forward into the second rope.

If the second rope is thicker (has more linear density) than the first rope, the incident wave will be partially reflected back with the reflection incurring a 180^{o} phase shift. But, because the amplitude of the reflected wave is less than the incident wave, the boundary between the two ropes will move up and down. That is the boundary will not be a node, a position of zero wave amplitude, as it was when the rope was attached to a rigid wall.

If the second roper is thinner (has less linear density) than the first rope, the incident wave will be partially reflected back without the reflection incurring a phase shift. However, the amplitude of the reflected wave will be less than the incident wave since some of the incident wave is transmitted on into the second rope.

The velocity of the transmitted wave in the second rope is different from the incident and reflected waves in the first rope. The velocity v of a wave traveling down a rope is equal to

$$v = \sqrt{\frac{F}{\mu}}$$

where

F = tension on the rope, and

μ = the density of the rope.

The tension F on both ropes is the same but their densities are not the same. Thus the velocity of wave propagation in the two ropes is different. A wave travels slower in a rope with high density $(larger\ \mu)$.

At the boundary of the two ropes, the frequency f of the wave transmitted into the second rope must be the same as that of the incident and reflected waves in the first rope. However, the wavelength of the wave in the second rope is different from the incident and reflected wavelengths in the first rope since wavelength

$$\lambda = \frac{v}{f}$$

Hence, the wavelength is shorter in a thicker rope where the wave velocity v is less.

4.4.2 Standing Wave

Suppose that two waves y_1 and y_2 of the same angular frequency ω, amplitude y_m , and speed are traveling in opposite directions along a string. The equations for the two waves are:

$$y_1 = y_m \sin(kx - \omega t)$$

and

$$y_2 = y_m \sin(kx + \omega t)$$

As expected, the only difference in the two equations is the sign preceding the angular frequency ω resulting from the waves traveling in opposite directions.

As before, the movement of each string particle is determined by the superposition of wave 1 and wave 2, that is by adding the two together which gives

$$y = y_1 + y_2 = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

Using trigonometric identities

$$\boldsymbol{y = 2y_m \sin kx \cos \omega t}$$

This is the equation for a standing wave.

Each particle in the string oscillates up and down in simple harmonic motion equal to cos ωt. However, the amplitude of oscillation, given by

$$y_x = 2y_m \sin kx$$

depends upon the position x of a particle along the string. Maximum amplitude, $2y_m$, occurs at positions where

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, etc$$

or

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, etc$$

remembering that

$$k = \frac{2\pi}{\lambda}$$

Points along the string where the amplitudes of oscillation are maximum are called antinodes. Notice that the antinodes are spaced one half wavelength apart along the string.

The amplitude of oscillation is zero at the positions where

$$kx = \pi, 2\pi, 3\pi, etc$$

or

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, etc$$

Particles along the string at these positions do not move, they are stationary. These points are called nodes and are also spaced one half wavelength apart.

The envelope waveform shown in Figure 16 illustrates the range of motion for each particle along the string.

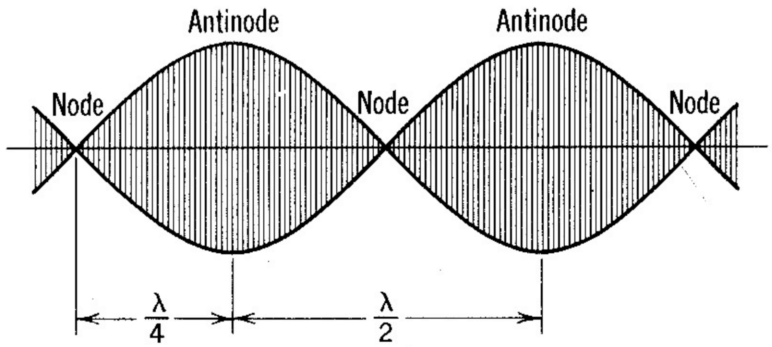


Figure 16 Standing Wave Envelope (source: Resnick & Halliday)

Note again that the string particles at the node positions do not move. They have zero amplitude. This is extremely important because it means that energy can not be transported down the string in either direction. Energy can not flow past nodal points in the string where the particles are permanently at rest.

Consequently, energy remains “standing” in the string. The energy of each string particle oscillates between potential energy, at the particle’s maximum displacement, and kinetic energy as the particle passes through zero displace heading toward its displacement in the opposite direction. However, the energy associate with the string particles can not travel to either the right or left along the string. It is “frozen” in place.

4.5 Resonance

The frequency at which a system oscillates in the absence of any driving force is called the system’s natural frequency. For example, in the spring mass system shown in Figure 17, if the mass m is pulled downward and then released, the system will oscillate on its own at a frequency ω_0 equal to

$$\omega_0 = \sqrt{\frac{k}{m}}$$

where

m = mass
k = spring stiffness.

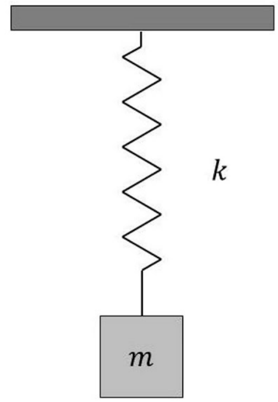


Figure 17 A Spring – Mass System

The motion of a system oscillating at its natural frequency is called its normal mode.

The system will oscillate at some other frequency if it is driven by a periodic external force. For example, if the structure supporting the spring mass system in Figure 17 were moved up and down at some periodic frequency. In this case the system oscillates at an amplitude that is generally equal to or less than that of the driving force.

The amplitude of the system's oscillation will become very large if the frequency of the external periodic force is equal to the system's natural frequency. This condition is know as resonance and the system is said to resonate with external force. In some cases resonance can cause the amplitude of the system's oscillations to become so great that the system is actually torn apart as in the Tacoma, Washington Bridge Disaster on November 7, 1940 shown in Figure 18.

Figure 18 Tacoma, Washington Bridge Disaster

As a further example, consider a string fixed at both ends, for example the string of a musical instrument. Standing waves can be established on the string provided that the end points of the string are both nodes. There can be any number of nodes in between or none at all. Consequently, a number of different standing waves can be induced on the string each with it own unique wavelength.

The distance between adjacent nodes is $\lambda/2$. So if the string's length is l, there must be exactly an integral number n of half wavelengths along the string's length. That is.

$$n\frac{\lambda}{2} = l \qquad n = 1, 2, 3, \cdots$$

The permitted wavelengths that can be supported on the string is thus

$$\lambda = \frac{2l}{n} \qquad n = 1, 2, 3, \cdots$$

The resonant frequencies at which the string can vibrate is determined from

$$\lambda = \frac{v}{f}$$

where

v = wave velocity, and
f = wave frequency.

Substituting in for wavelength

$$\lambda = \frac{v}{f} = \frac{2l}{n} \qquad n = 1, 2, 3, \cdots$$

$$f = \frac{nv}{2l} \qquad n = 1, 2, 3, \cdots$$

The velocity of a wave traveling down a stretched string is

$$v = \sqrt{\frac{F}{\mu}}$$

where

F = the tension or force exerted on the string, and
μ = the mass per unit length of the string

The permitted frequencies of string oscillation are therefore

$$f_n = \frac{n}{2l}\sqrt{\frac{F}{\mu}} \qquad n = 1, 2, 3, \cdots$$

These are the string's natural frequencies.

If a periodic driving force near any of the string's natural frequencies is applied to the string, for example a violin bow moved across one of the violin's strings, the string will vibrate at that frequency with a large amplitude. Because the string has a large number of natural frequencies, resonance will occur at any of these frequencies. This is in contrast to the spring mass system where there is only one resonant frequency.

5 Longitudinal Waves

As discussed earlier, a longitudinal wave is characterized by particles within a medium oscillating back and forth in the same direction as a wave traveling through the medium. Consequently zones of particle compression (particles squeezed closer together) and rarefication occur in longitudinal waves as shown in Figure 19.

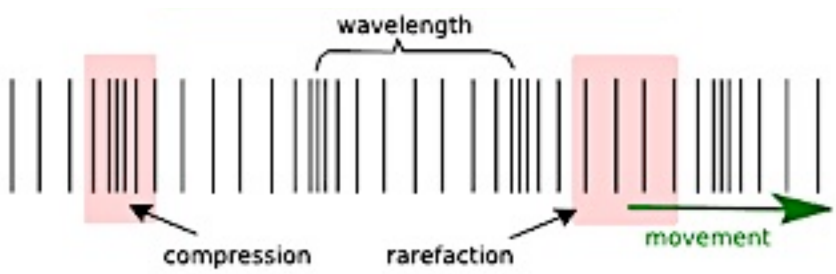


Figure 19 A Longitudinal Wave

5.1 Sound Waves

Sound waves are longitudinal mechanical waves. Sound waves can propagate through gases, liquids, and solids. As a wave passes, the particles composing a material vibrate back and forth in the same direction that the wave is traveling.

Sound waves in the frequency range from approximately 20 Hz to 20,000 Hz (20 KHz) are defined as audible sound waves since they stimulate the human ear and brain producing the sensation of hearing. The range of frequencies that animals can hear varies widely. For example, dogs can hear sound waves that are much higher in frequency than what humans can detect. This has given rise to dog whistles used in calling dogs. While people can not hear the whistle, dogs hear it clearly.

Sound waves at frequencies greater than 20 KHz are called ultrasonic waves. The frequency of ultrasonic waves range up to 600 MHz. These extremely high frequency mechanical waves are produced by the elastic vibrations of very thin quartz crystal wafers when an alternating electric field is placed across the wafer.

Sound waves at frequencies below 20 Hz are called infrasonic waves. These very low frequency sound waves are usually generated by large physical events such as earthquakes.

Audible sound waves are typically produced by:

- **Vibrating strings**: For example the sounds produced by the vast assortment of stringed musical instruments (violins, guitars, harps, etc.) and also by human vocal cords.
- **Vibrating columns of air**: Examples include organs, clarinets, trumpets, saxophones, etc.
- **Vibrating plates and membranes**: For example drums, xylophones, and loudspeakers.

In each case, the vibrating elements alternately compresses the surrounding air as the element moves forward and rarefies the air as it moves backward, producing a longitudinal wave that propagates outward through the air from the vibrating element.

Sound waves which are approximately periodic or composed of a small number of periodic components are perceived by the human ear-brain as a pleasant sound, for example a human voice or musical sounds. Sounds with very irregular waveforms are interpreted by the human ear – brain system as unpleasant noise.

5.2 Propagation of Longitudinal Waves

If unimpeded, a longitudinal wave, such as a sound wave, will travel radially outward forming spherical wave fronts centered on the source of the wave as illustrated in Figure 20. However, in general the propagation of a longitudinal wave is constrained, often in complex ways, by physical objects in the vicinity of the wave's source.

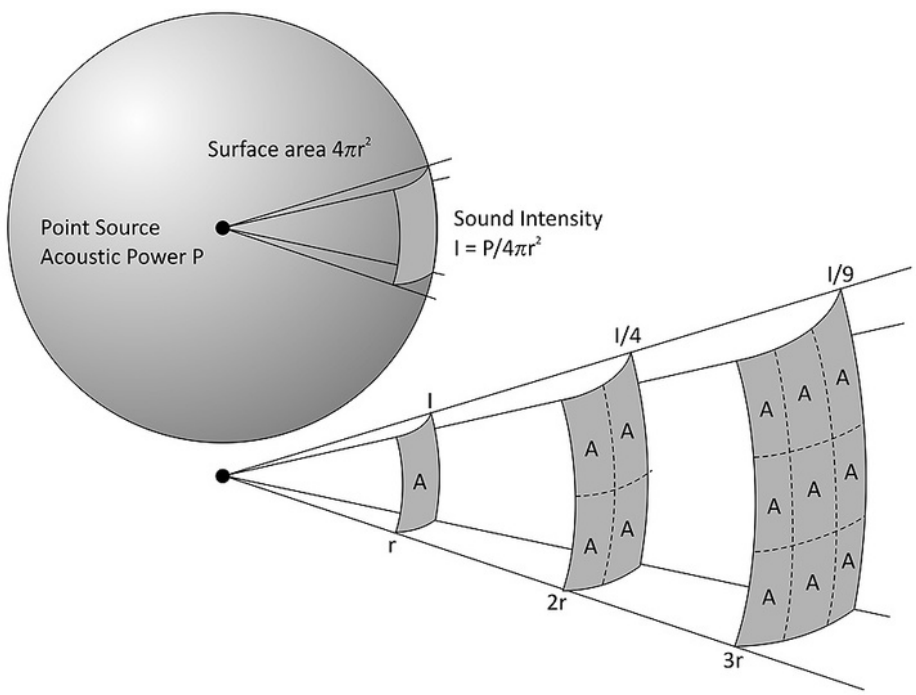


Figure 20 Spherical Waves (source soundonsound.com)

In Figure 21 a one dimensional longitudinal wave, moving to the right, is constrained in its movement by a long tube. The tube is filled with a compressible fluid through which the wave propagates. The tube could be a water pipe. Longitudinal waves move down a water pipe when the water is first turned on. The nature of the wave movement depends upon whether the far end of the pipe is open, closed, or partially closed. A water pipe that has air trapped in the pipe can shake violently when water is first turned on. A sound wave is generally produced if the pipe is filled with air.

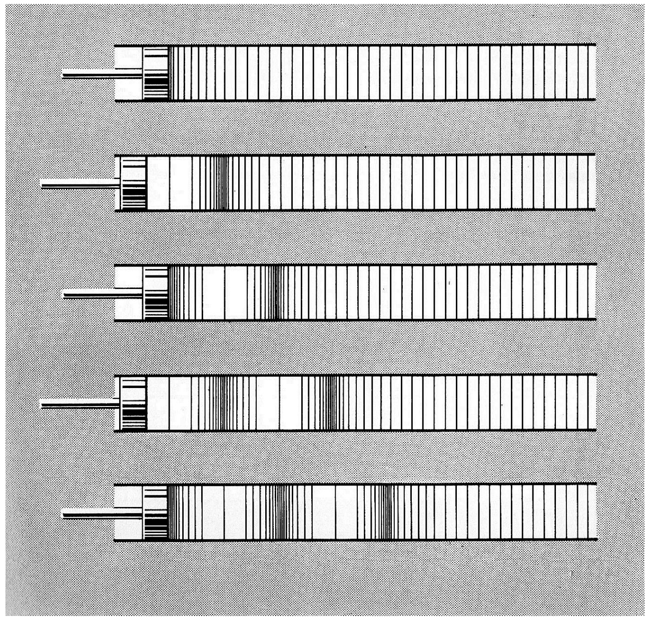


Figure 21 Sound wave generated by oscillating piston (source: Resnick & Halliday)

The longitudinal wave in Figure 21 is produced by an oscillating piston at the left end of the tube. The vertical lines within the tube represent fluid density. The vertical lines to the right of the piston in the top illustration are all evenly spaced indicating that fluid density throughout the tube is uniform. That is, the fluid in the tube is undisturbed. When the piston moves quickly forward (to the right) it compresses the fluid directly in front of it. The compressed fluid in turn compresses the fluid to its right and so on causing a compression wave front to move down the length of the tube. As this is happening the piston quickly reverses direction moving backwards rarefying the fluid density as shown in the second illustration. This region of rarefication also travels to the right down the tube. The piston continues moving back and forth at a specific frequency producing longitudinal waves moving to the right down the length of the tube. The physical distance from one region of compression to the center of the next compression is the wavelength λ of the longitudinal wave. The time period T between the formation of one compression region and formation of the following compression is the period of the wave. The wave's frequency f is the same as the frequency at which the piston is oscillating back and forth, that is

$$f = \frac{1}{T}$$

5.3 Speed of Longitudinal Wave Propagation

The speed v of a longitudinal wave traveling through a fluid is equal to

$$v = \sqrt{\frac{B}{\rho_0}}$$

where

ρ_0 = the density of the undisturbed fluid

B = the bulk modulus of elasticity of the fluid

B is the ratio of pressure change, ΔP , on a fluid element to the fractional change in the element's volume $-\Delta V/V$. That is

$$B = -\frac{V}{\Delta V}\Delta P$$

Notice that the speed of a longitudinal wave through a fluid is determined by the fluid's properties.

If the fluid happens to be a gas, such as air, the longitudinal wave is typically a sound wave traveling through the gas at a speed of

$$v = \sqrt{\frac{\gamma P_0}{\rho_0}}$$

where

γ = the ratio of specific heats for the gas
P_0 = the undisturbed gas pressure
ρ_0 = the undisturbed gas density

In this case the bulk modulus of elasticity B is equal to

$$B = \gamma P_0$$

The ratio of specific heats γ is

$$\gamma = \frac{C_p}{C_v}$$

where

C_p = Specific heat at constant gas pressure, and

Cv = Specific heat at constant gas volume

Specific heat is the heat per unit mass required to raise a substance's temperature by one degree K.

The speed of sound through various substances is show in Table 1.

Medium	Temperature ℃	Meters / second
Oxygen	0	317.2
Air	0	331.3
Hydrogen	0	1,286.0
Water	15	1,450.0
Lead	20	1,230.0
Copper	20	3,560.0
Aluminum	20	5,100.0
Iron	2o	5,130.0

Table 1 Speed of Sound in Various Medium

5.4 Equations of Motion for Longitudinal Waves

In Figure 21 it is presumed that the x-axis extends down the center of the tube. A particle of fluid oscillates back and forth in the plus and minus x direction about its equilibrium position at some location x as a longitudinal wave propagates to the right (in the +x direction) down the tube. Thus the oscillations of a fluid particle and the direction of wave travel are both along the x-axis.

For consistency with transvers wave equations, we let y represent the displacement along the x-axis of a fluid particle from its equilibrium position. However, in this case the displacement y is along the same axis as the direction of wave travel. In contrast the displacement y for a transverse wave is perpendicular to the direction of wave propagation.

The equation for a longitudinal wave traveling to the right down the tube is

$$y = f(x - vt)$$

where y is the displacement of a particle at time t from its equilibrium position at some point x and v is the velocity of the wave. Thus the actual displacement y is a function f of x, v, and t. For the case of a sinusoidal wave, the simple harmonic motion of each particle is given by

$$y = y_m \cos\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

where

y_m = wave amplitude = maximum displacement of a fluid particle

λ = wavelength

As with transverse waves, the equation can be written as

$$y = y_m \cos(kx - \omega t)$$

by defining

$$k = \frac{2\pi}{\lambda}$$

In practical applications it is usually more meaningful to deal with pressure variations caused by a longitudinal wave than with the displacement of fluid particles.

As described above, the bulk modulus of elasticity B is equal to

$$B = -\frac{V}{\Delta V}\,\Delta P$$

so the change in pressure ΔP is equal to

$$\Delta P = -B\frac{\Delta V}{V}$$

If we define P to be the change from the undisturbed pressure P_0 then

$$P = \Delta P = -B\frac{\Delta V}{V}$$

If an element of fluid at pressure P_0 has a length Δx and a cross sectional area A, then its volume is

$$V = A\,\Delta x$$

A pressure change changes the volume of an fluid element by

$$\Delta V = A\,\Delta y$$

where Δy is the amount by which the length of the element changes. Thus

$$P = -B\frac{\Delta V}{V} = -B\,\frac{A\,\Delta y}{A\,\Delta x}$$

If the length Δx becomes very small, that is $\Delta x \rightarrow 0$ then

$$P = -B\frac{dy}{dx}$$

If the particle displacement is sinusoidal such that

$$y = y_m\cos(kx - \omega t)$$

then

$$\frac{dy}{dx} = -ky_m\sin(kx - \omega t)$$

and the change in pressure is

$$P = Bky_m\sin(kx - \omega t)$$

However,

$$v = \sqrt{\frac{B}{\rho_0}}$$

so

$$B = v^2\rho_0$$

which gives the following equation for a pressure wave

$$P = [k\rho_0 v^2 y_m]\sin(kx - \omega t)$$

The term

$$P_A = [k\rho_0 v^2 y_m]$$

is the pressure wave amplitude. The equation for a pressure wave can then be written simply as

$$P = P_A \sin(kx - \omega t)$$

Consequently a longitudinal wave in a fluid can be considered to be either a displacement wave or a pressure wave. The difference is that a displacement wave

$$y = y_m \cos(kx - \omega t)$$

is 90^{o} out of phase with its corresponding pressure wave

$$P = P_A \sin(kx - \omega t)$$

When the displacement of a particle is zero, $y = 0$, the pressure difference $P - P_0$ is maximum at that point. Furthermore, when the particle displacement is maximum, $y = y_m$, the pressure difference is zero.

5.5 Standing Longitudinal Waves

A longitudinal wave traveling within a tube is reflected at the end of the tube in a manner similar to the reflection of a transverse wave when it reaches the end of a rope. A standing longitudinal wave results from the interference between the incident wave traveling in one direction and the reflected wave traveling in the opposite direction.

The reflected wave is 180^{o} out of phase with the incident wave if the end of the tube is closed. This must be the case since the displacement of particles at the closed end of the tube is always zero. The closed end of the tube is thus a displacement node. However, particles are free to move at the end of an open tube. The reflections at the end of an open tube depend on wavelength. The reflected wave has nearly the same phase as the incident wave if the tube is narrow compared to the wave's wavelength. This is the case in most musical instruments. The end of an open tube is almost a displacement antinode. The exact position of the antinode is usually somewhere near the opening. However, the length of an air column in a musical instrument is not as well defined as the string length of a stringed musical instrument in which the string is fixed at both ends.

6 Vibrating Systems and Sources of Sound

The strings of a violin are similar to those of most stringed instruments. Both ends of a violin string are anchored in place, first by the bridge and tail piece at the bottom of the violin and second by the peg box at the upper end of the violin as shown in Figure 22. The length of the string is shortened by the violinist's fingers pushing the string against the finger board. Transverse waves are induced into the string as the violinist moves the violin bow perpendicularly across the string. The induced waves are reflected at the fixed ends of the string forming standing waves along the string's length. The oscillating string creates longitudinal waves in the surrounding air that propagate outward reaching our ears as a musical note.

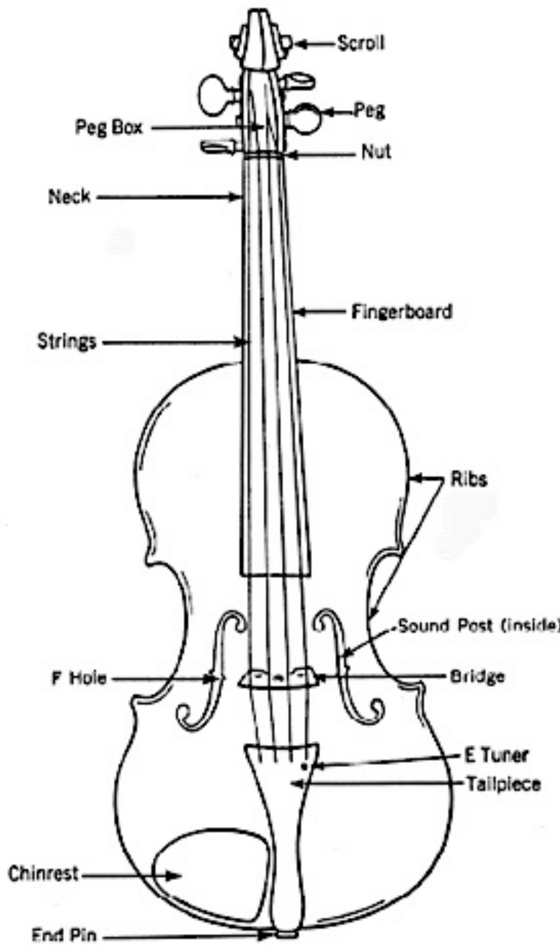


Figure 22 Violin Diagram (source Pinterest)

As discussed earlier, a violin string of length l can resonate (form standing waves) at a number of different frequencies given by

$$f_n = \frac{n}{2l}\sqrt{\frac{F}{\mu}} \qquad n = 1, 2, 3, \cdots$$

The string will contain an integer number n of loops along its length at any of these frequencies as illustrated in Figure 23.

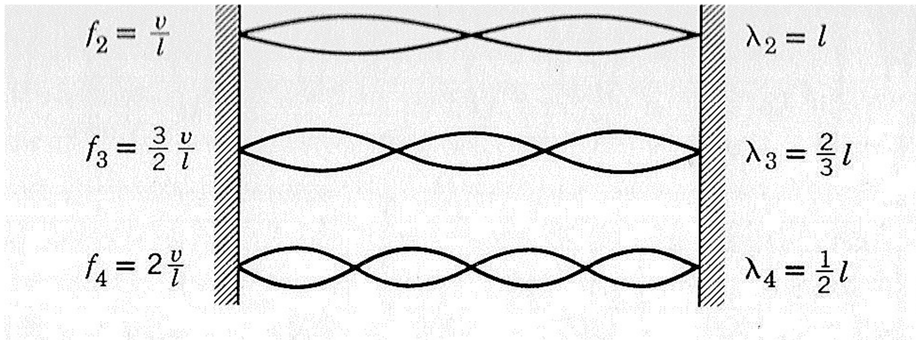


Figure 23 First four modes of vibration of a string fixed at both ends (source: Resnick & Halliday)

The lowest frequency

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$$

is called the fundamental frequency and the higher frequencies are called overtones or harmonics. The fundamental is the first harmonic. The frequency $f_2 = 2 * f_1$ is the first overtone or the second harmonic. Likewise, the frequency $f_3 = 3 * f_1$ is the second overtone or the third harmonic.

A particular note is played on a violin by drawing the bow across a violin string causing the string to vibrate at its fundamental frequency, the frequency of the desired note. In addition to vibrating at this frequency, the string also vibrates at many of its overtone frequencies as well. It is these overtones, each with their own amplitudes, that combine with the fundamental frequency to give the violin its quality of tone. The actual displacement of the violin string is the sum of the displacements caused by the fundamental and the various overtones as illustrated in the top of Figure 24. The figure also shows the frequency and relative amplitude of the fundamental and the predominant overtones. In this case the fundamental frequency is 440 Hertz, the note “A” on the musical scale. Since the fundamental frequency is the reference, its relative amplitude must be 1.0. Notice that both the first and fourth overtones are particularly strong.

Figure 24 also shows the displacement of a piano string when the note “A” is played along with the frequencies and relative amplitudes of its overtones. Notice how small the overtone amplitudes are for the piano compared to the violin. Consequently, the note “A” played on a piano is very pure, nearly a sinusoid at 440 Hertz.

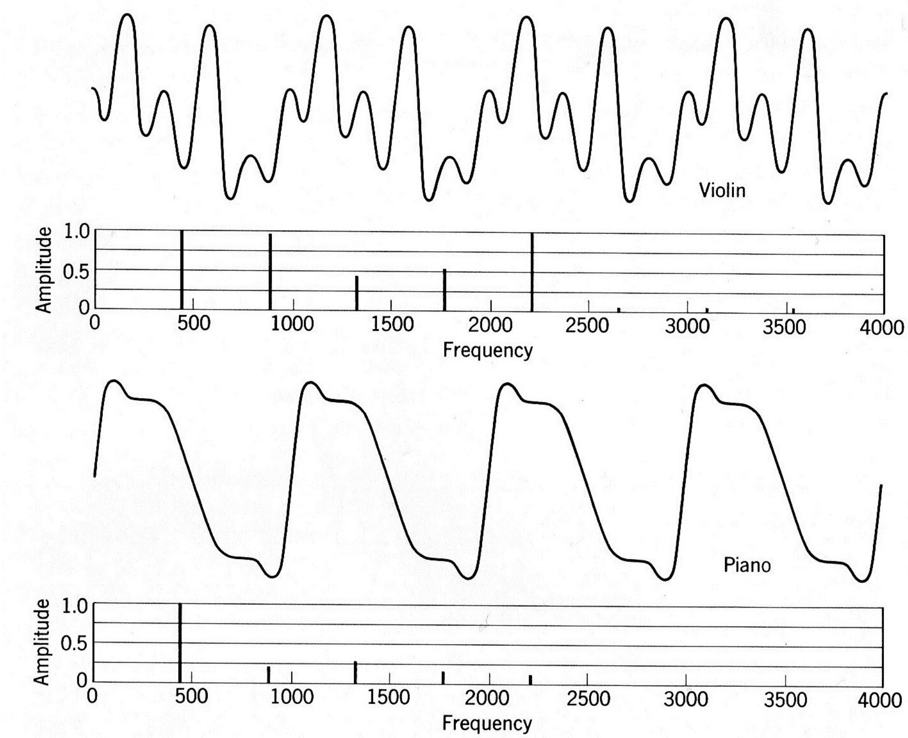


Figure 24 Wave form and spectrum for a violin and piano playing the same note (source: Resnick & Halliday)

The sound produced by a vibrating column of air in an organ pipe illustrates the basic mechanism for generating musical sounds in all wind instruments. The diagram of a typical organ pipe is shown in Figure 25. The top of the pipe is open while air is introduced through an opening at the bottom of the pipe. The air is deflected by the Languide across a small opening called the Mouth creating a disturbance at the bottom of the tube. The disturbance causes the column of air in the tube to resonate at its various natural frequencies given by

$$f_n = \frac{n}{2l} v \qquad n = 1, 2, 3, \cdots$$

where

l = the length of the organ pipe = L in Figure 25
n = the integral number of wave harmonics and
v = the speed of longitudinal waves in the column.

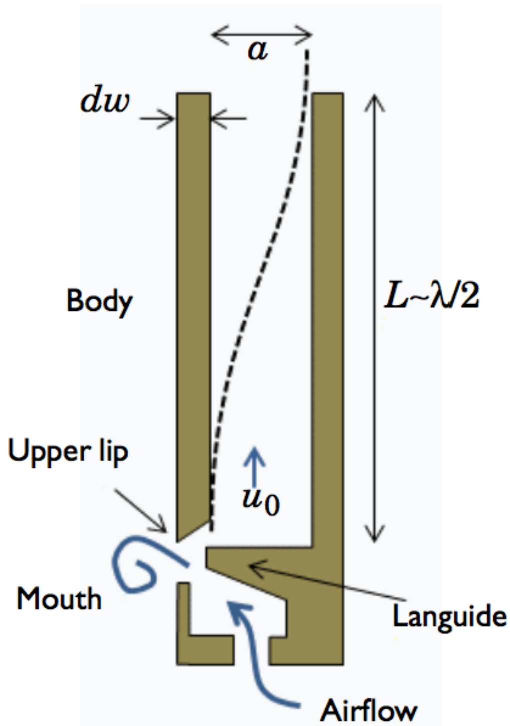


Figure 25 Diagram of an organ pipe (source: Comsol)

The fundamental frequency (n = 1) and the harmonics (n > 1) are excited at the same time. In an open pipe the fundamental frequency produces a displacement antinode at each end of the pipe and a displacement node in the middle as shown in Figure 26. The figure also shows the first three harmonics , the second (1st overtone), third (2nd overtone), and fourth harmonic (3rd overtone). Thus in an open organ pipe the fundamental frequency is approximately

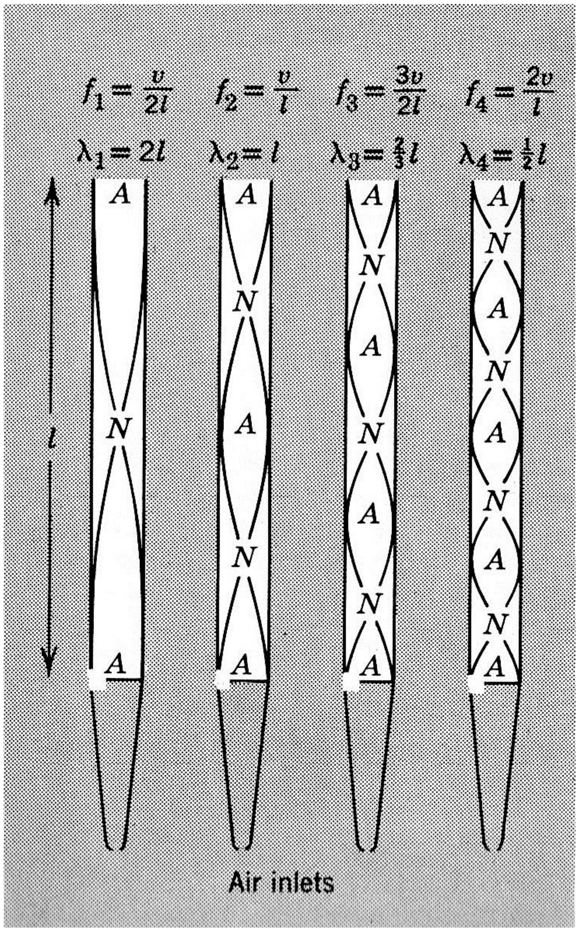
$$f_1 = \frac{1}{2l} v$$

and all harmonics are present.

In a closed pipe the closed end must be a displacement node as shown in Figure 27. The air inlet at the opposite end of the pipe is open and must be a displacement antinode. In this case the fundamental frequency is approximately

$$f_1 = \frac{1}{4l} v$$

which is half that of an open pipe of the same length. The only harmonics possible in a closed pipe are those that produce a displacement node at the closed end and an antinode at the open end. Consequently, the even harmonics (2nd harmonic, 4th , etc.) are missing. In a closed pipe only the fundamental frequency and the odd harmonics (3rd, 5th, etc.) are present. Consequently, the sound from an open pipe is quite different from that of a closed pipe.



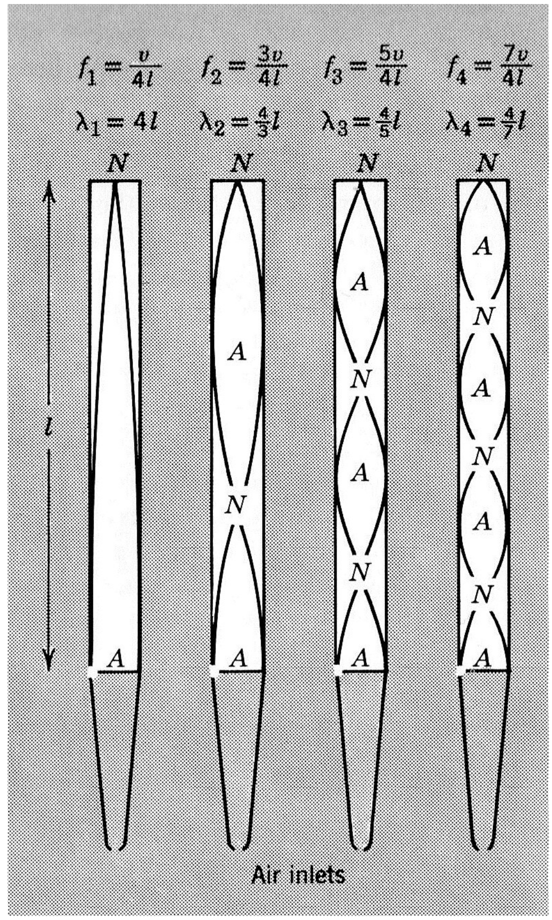


Figure 26 Open Organ Pipe
(source: Resnick & Halliday)

Figure 27 Closed Organ Pipe
(source: Resnick & Halliday)

7 Beats

Beats are variations in loudness that occur when two sound waves of equal amplitude, but slightly different frequency, travel through the air at the same time. Beats occur, for example, when two adjacent piano keys are struck simultaneously. Beats can also occur for other types of longitudinal waves and in various types of fluid.

Figure 28a shows two sound waves with the same amplitude and slightly different frequencies. The displacement of air particles is the sum of the displacements due to each wave and is shown in Figure 28b. Usually the human ear-brain can hear each of the original sound waves plus the resulting wave, as for example when striking two adjacent piano keys. Notice that the amplitude (loudness) of the resulting sound wave increases to a maximum, decreases to zero, and then increases again, while the loudness of the two original waves remain constant. Beats are denoted as the points in time when the resultant sound wave reaches maximum amplitude.

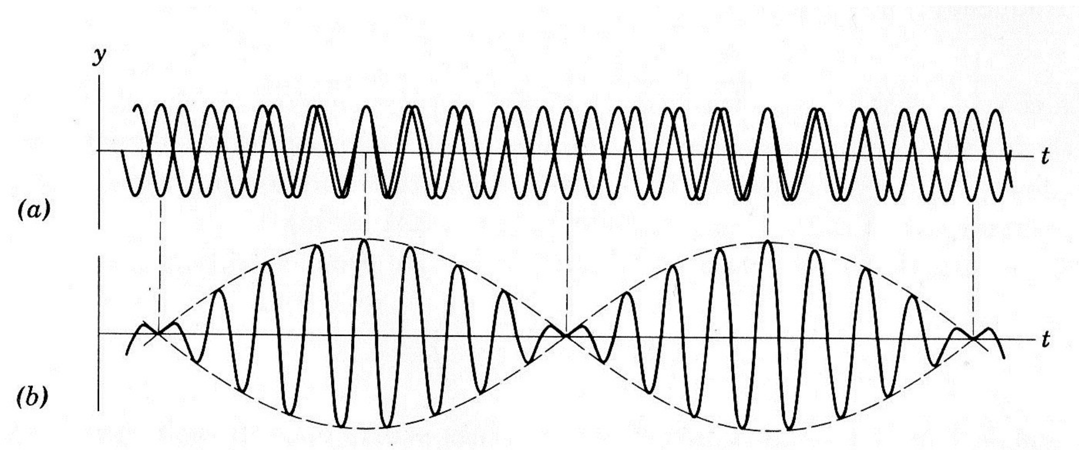


Figure 28 Beat Phenomenon (source: Resnick & Halliday)

Beats are commonly used for tuning musical instruments. Two strings may be tuned to the same frequency by tightening one of the strings until the beat between the two strings disappears. When the two strings are at the same frequency, the amplitude of the resulting sound is constant (instead of varying) and twice as loud as either string by itself as illustrated in Figure 29. In this figure the sounds produced by the individual strings are the green and blue waves. The red wave is the resulting sound wave.

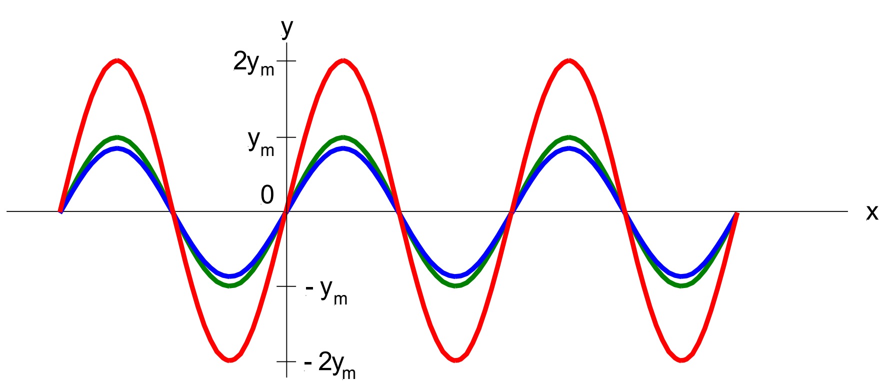


Figure 29 Resulting sound (red) from two strings (green and blue)

The frequency and amplitude of the resulting wave shown in Figure 28b is determined as follows.

The displacement of air particles caused by the first wave is equal to

$$y_1 = y_m \cos 2\pi f_1 t$$

and that caused by the second wave is

$$y_2 = y_m \cos 2\pi f_2 t$$

Adding the two displacements together gives the resulting displacement

$$y = y_1 + y_2 = y_m(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

Since

$$\cos a + \cos b = \left[2\cos\frac{a-b}{2}\cos\frac{a+b}{2}\right]$$

then

$$y = \left[2y_m \cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

The sinusoidal oscillations of the resulting wave are

$$\cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

with a frequency

$$f_\alpha = \left(\frac{f_1 + f_2}{2}\right)$$

which is the average of the original two wave frequencies.

The amplitude of the resulting wave is the term in brackets, which is

$$2y_m \cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t$$

This is the amplitude of the wave envelope in Figure 28b. Consequently, the amplitude of the resulting wave, and thus the wave envelope, varies sinusoidally at a frequency of

$$f_\delta = \frac{f_1 - f_2}{2}$$

which is half the difference between the frequencies of the original two waves. The frequency of the wave envelope will be a very low if f1 and f2 are nearly equal, that is, the amplitude of the resulting wave will vary slowly.

If the frequencies of the original two waves are equal, that is $f_1 = f_2$, then the equation

$$y = \left[2y_m \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t\right] \cos 2\pi \left(\frac{f_1 + f_2}{2}\right) t$$

becomes

$$y = \left[2y_m \cos 2\pi \left(\frac{0}{2}\right) t\right] \cos 2\pi \left(\frac{2f_1}{2}\right) t$$

which reduces to

$$y = 2y_m \,(\cos 2\pi f_1)\, t$$

as illustrated in Figure 29. In this case the frequency of the wave envelope has gone to zero meaning that the amplitude of the wave envelope has become a constant at a value of $2y_m$, again as shown in Figure 29.

Returning to the case where the two frequencies f_1 and f_2 are slightly different, the beat, defined as the maximum amplitude of the resulting wave, will occur whenever

$$\cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \qquad equals\ 1\ or - 1$$

Cosine $\beta = 1$ when

$$\beta = 0, \quad 2\pi, \quad 4\pi, \quad \cdots$$

and equals -1 when

$$\beta = \pi, \quad 3\pi, \quad 5\pi, \quad \cdots$$

Consequently, the amplitude of the resulting wave is maximum when

$$t = 0, \quad \frac{2}{f_1 - f_2}, \quad \frac{4}{f_1 - f_2}, \quad \cdots$$

and

$$t = \frac{1}{f_1 - f_2}, \quad \frac{3}{f_1 - f_2}, \quad \frac{5}{f_1 - f_2}, \quad \cdots$$

These conditions occur twice in each cycle so there are two beats per cycle. That is, the amplitude of the resulting wave reaches a maximum value twice in each cycle, once a positive maximum and the other a negative maximum. The frequency of beats is thus

$$2f_\delta = \frac{f_1 - f_2}{2} = f_1 - f_2$$

or at a frequency equal to the difference between the frequencies of the original waves.

8 Doppler Effect

The sound of a ambulance siren seems higher in frequency to a stationary listener when the ambulance is approaching than when it is standing still. In contrast, the frequency of the siren sounds lower when the ambulance is moving away. These phenomena are know as the Doppler effect. Intuitively, the sound waves from an approaching ambulance are compressed making them sound higher in frequency. When the ambulance is moving away, the sound waves are stretched out causing them to sound lower in frequency as illustrated in Figure 30.

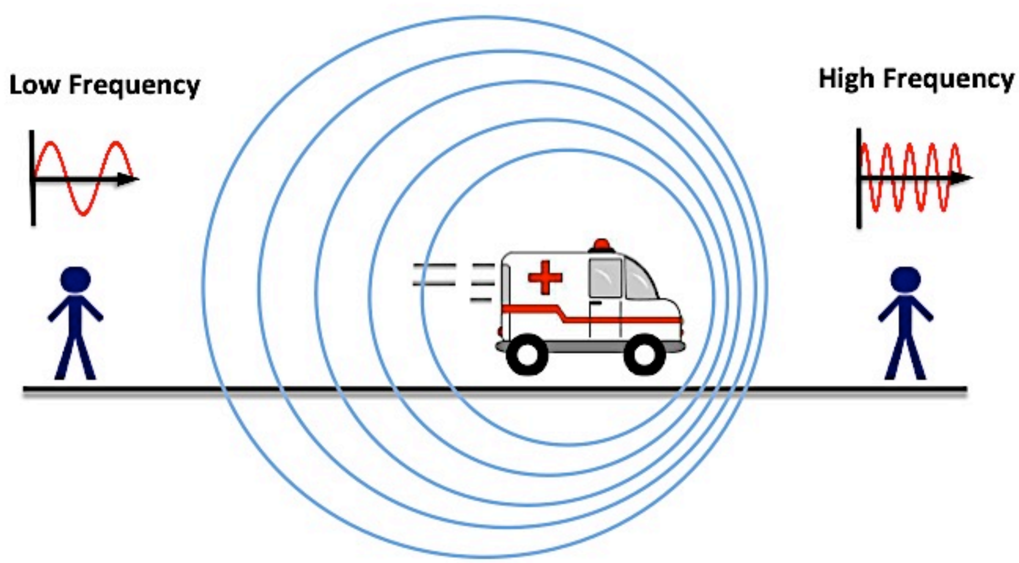


Figure 30 Doppler Effect (source: Pinterest)

Our intuition can be easily verified.

f_{LSA} = frequency of a sound when the source of the sound is approach (SA) a stationary listener (L).

f_S = frequency of a sound wave when the source (S) of the sound is at rest

λ_S = wavelength of a sound wave when the source is at rest

T_S = period of a sound wave with the source is at rest

v_S = speed of the sound source

v = speed of sound through the air

The period of a sound wave when the source is at rest is

$$T_S = \frac{1}{f_S}$$

The wavelength of the sound wave with the source at rest is

$$\lambda_S = v\,T_s = \frac{v}{f_S}$$

The speed of sound through air, v , is a property of the air through which the sound is traveling and does not depend on the speed at which the source of the sound is moving.

In the time period T_S the source travels toward the listener a distance of

$$d = v_S\,T_S$$

Consequently, the wavelength of the sound appears to be short than it actually is by an amount equal to d . Thus the wavelength perceived by the listener is

$$\lambda_{LSA} = \lambda_S - d = \frac{v}{f_S} - v_S\,T_S = v\,T_S - v_S\,T_S = (v - v_S)T_S = \frac{v - v_S}{f_S}$$

Since the speed of sound v through the air does not depend on the speed of the source, the frequency of the sound as perceived by the listener is

$$f_{LSA} = \frac{v}{\lambda_{LSA}} = \frac{v\,f_S}{(v - v_S)} = f_S\left[\frac{v}{v - v_S}\right]$$

which is higher in frequency than the sound actually produced by the sound source.

If the sound source is moving away from the listener (source receding), the distance d is **added** to the sound's wavelength giving

$$\lambda_{LSR} = \lambda_S + d = v\,T_S + v_S\,T_S = (v + v_S)T_S = \frac{v + v_S}{f_S}$$

and the frequency of the sound heard by the listener is

$$f_{LSR} = \frac{v}{\lambda_{LSR}} = \frac{v\,f_S}{(v + v_S)} = f_S\left[\frac{v}{v + v_S}\right]$$

which is lower in frequency than the actual sound emitted by the sound source.

For a listener approaching a stationary sound source the situation is slightly different.

In this case the speed of the listener as he or she approaches the sound source is v_L . Again, the speed of sound v through the air is determined by the characteristics of the air and has nothing to do with the speed of the listener. The effective speed of the sound wave as it reaches the moving listener is

$$v_a = v + v_L$$

The period of the sound wave perceived by the approaching listener is

$$T_{AL} = \frac{\lambda_S}{v_a} = \frac{\lambda_S}{v + v_L}$$

The frequency of the sound as heard by the approaching listener is

$$f_{AL} = \frac{1}{T_{AL}} = \frac{v + v_L}{\lambda_S}$$

and since

$$\lambda_S = \frac{v}{f_S}$$

$$f_{AL} = \frac{1}{T_{AL}} = \frac{v + v_L}{\lambda_S} = \frac{v + v_L}{v/f_S} = f_S\left[\frac{v + v_L}{v}\right]$$

which is higher in frequency than the sound actually produced by the sound source.

The speed of the listener is subtracted from the speed of sound if the listener is moving away (receding) from the sound source. In this case the sound heard by the listener is

$$f_{RL} = \frac{1}{T_{RL}} = \frac{v - v_L}{\lambda_S} = \frac{v - v_L}{v/f_S} = f_S\left[\frac{v - v_L}{v}\right]$$

which is lower in frequency.

9 Shock Wave

A shock wave is produced when a disturbance is moving faster through a medium than information related to the disturbance.

For example, in Figure 31 an aircraft flying faster than the speed of sound passes overhead before we hear it. After the aircraft has passed we hear a loud sonic boom followed by the sound of the aircraft. In this example, information concerning the presence of the aircraft, the sound of its engines, reaches us after the aircraft has already gone by. In contrast, we hear an aircraft approaching long before it passes overhead if it is flying at less than the speed of sound. In this case, information concerning the presence of the aircraft, the sound of its engines, travels faster than the aircraft and reaches us before the aircraft itself.

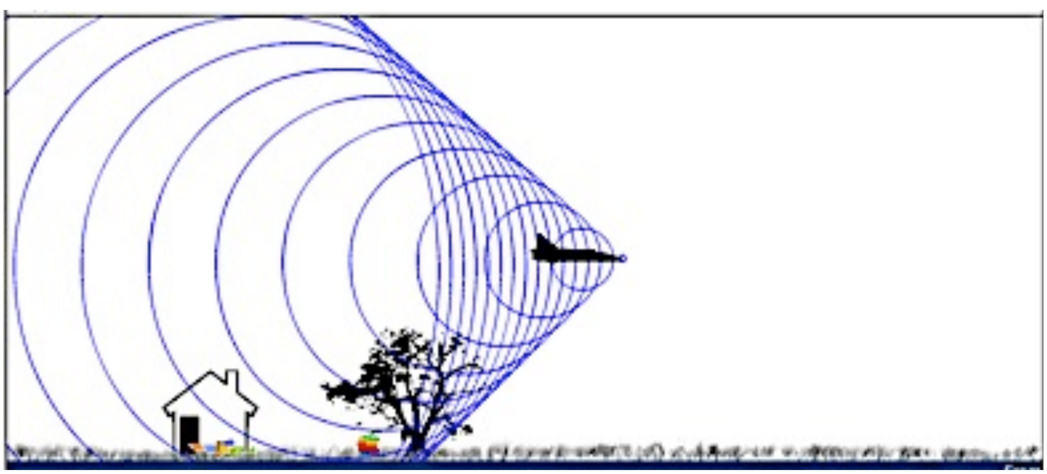


Figure 31 Sonic Boom created by aircraft traveling at faster then speed of sound (source: Physclips)

In Figure 32 an aircraft is standing still (hovering some how). The sound made by its engines expands outward in equally spaced spherical wave fronts. The blue wavefront was produced some time earlier at time t_0 and has expanded further then the other wavefronts, while the cyan wavefront was recently produced at time $t_n > t_0$. As the aircraft moves forward (to the right) the wavefronts are compressed in the aircraft's direction of travel as shown on the left in Figure 33. The wavefronts are compressed to the point where they all over lap at the position of the aircraft as the aircraft reaches the speed of sound. The expansion of wavefront t_0 can not keep up with the speeding aircraft when the aircraft travels faster than the speed of sound. That is, the spherical wavefront t_0 falls behind the aircraft as do all of the other wavefronts. When this happens the wavefronts form a cone extending out behind the aircraft creating a sonic boom as illustrated on the right in Figure 33.

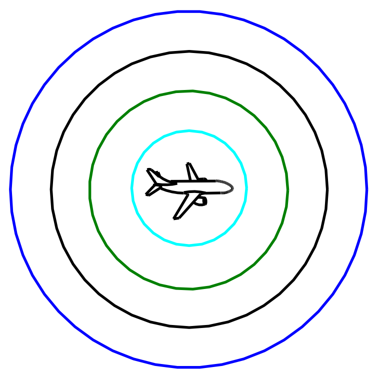
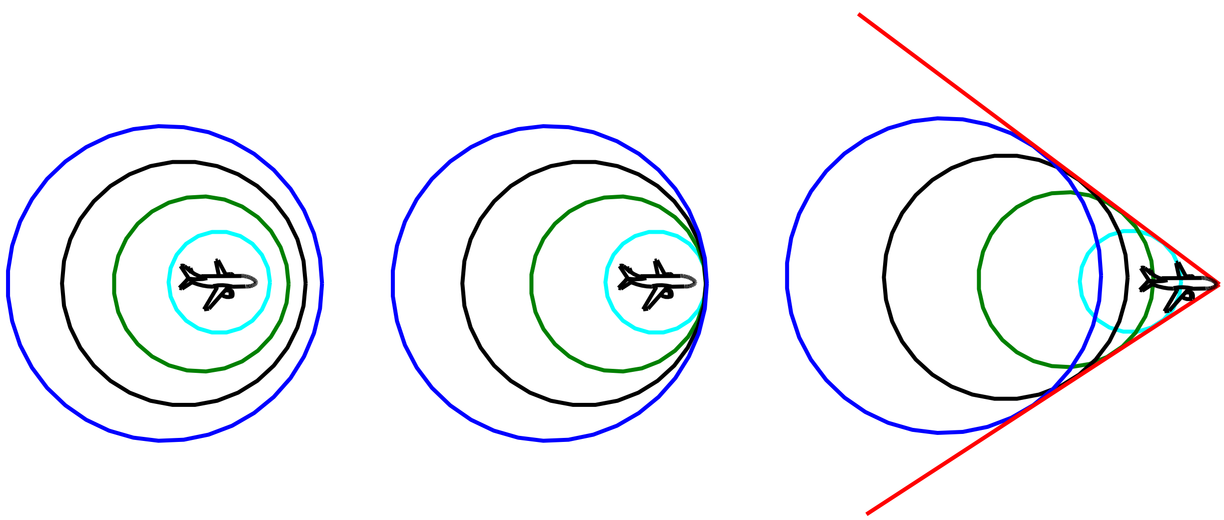


Figure 32 Aircraft Hovering (standing still)



Speed: Mach < 1 Speed: Mach = 1 Speed: Mach > 1

Figure 33 Aircraft moving forward at progressively higher speeds

The speed of an aircraft is often expressed in Mach numbers.

$$Mach = \frac{v_a}{v}$$

where

v_a = speed of the aircraft

v = speed of sound

At sea level with an air temperature of 20° C the speed of sound v is 1,225 km/hr or 761 miles per hour. An aircraft traveling at 1,000 mph is thus traveling at

$$Mach = \frac{v_a}{v} = \frac{1{,}000}{761} = 1.3$$

In more general terms the Mach number for a disturbance moving through some medium is

$$Mach = \frac{v_s}{v}$$

where

v_s = speed of the disturbance

v = speed of wave propagation through the medium

Figure 34 illustrates the shock wave phenomena in more detail. At time t_0 a supersonic aircraft is at position S'. After a time t the sound from the aircraft has expanded outward from position S' a distance of vt , where v is the speed of sound, forming the largest circle (wavefront) shown in Figure 34. During that same time t the aircraft, moving at a speed of v_s , has traveled a distance of $v_s t$ arriving at position S . The aircraft has clearly outrun the wavefront formed when it was at position S' . The same is true of all the wavefronts shown in Figure 34. The envelope of these wavefronts is a cone whose surface makes an angle of θ with the aircraft's line of travel from S' to S . The cone is the shock wave created by the aircraft traveling at faster than the speed of sound. The width of the shock wave cone depends on the aircraft's speed.

From Figure 34 we see that

$$\sin\theta = \frac{v}{v_s} = \frac{1}{Mach\ Number}$$

Consequently, the higher the aircraft's speed v_s (the greater its Mach number) the narrower the cone.

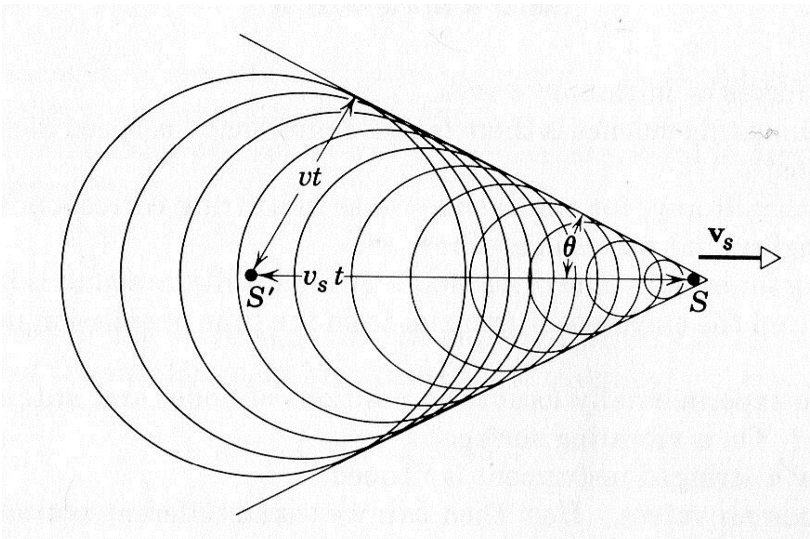


Figure 34 Shock Wave (source: Resnick & Halliday)

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