Determining the Speed of Light

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3 Speed of Light

The speed of light, c, is quite possibly the most important fundamental constant in all of physics. It relates the mass of a particle to its energy in Einstein's equation

$$E = mc^2$$

The Special Theory of Relativity states that the speed of light, c, is the maximum speed at which an object can travel, and that strange things, like time dilation, occur as the speed of light is approached. Today, a number of other important physical constants, like the length of a meter, are defined in terms of c. The speed of light is used to measure an aircraft's altitude on final approach to landing, and to measure the distance to other planets, other galaxies, and the size of the Universe itself. It enters into the conversion between electrostatic and electromagnetic units. In radio technology the speed of light is critically important since, among other things, it determines the length of a transmitting antenna. That is, antenna length

$$L = \frac{1}{n} \cdot \frac{c}{f}$$

where f is the frequency of the transmitted signal and n is an integer typically 1, 2, or 4. For example, L is equal to the length of a half wavelength $\left(\frac{1}{2}\lambda\right)$ antenna if n = 2.

Because of its importance, more time and effort have been devoted to the measurement of c than any other physical constant. Despite this enormous effort, an accurate value for the speed of light has just recently been obtained.

3.1 Early Conjecture Concerning the Speed of Light

The speed of light, whether infinite (instantaneous) or finite, was debated for over two thousand years.

In the fifth century BC the Greek philosopher Empedocles (~ 492 – 432 BC) postulated that everything was composed of four elements: fire, air, earth, and water. Empedocles believed that sight was the result of light emitted from our eyes that bounced off objects and back into our eyes permitting us to see an object's size, shape and color. He was also the first recorded philosophers to speculate that light traveled at a finite speed. He reasoned that light was something in motion and therefore must take some time to travel.

However, Aristotle (384 – 322 BC) argued that light was due to the presence of something, not a movement as Empedocles suggested. Thus, he argued that light appears instantly. Because of his reputation, most philosophers and early scientists followed Aristotle's teaching believing that the speed of light is infinite.

Euclid (~ 325 – 265 BC) lived in Alexandria, Egypt and studied at the great library of Alexandria shortly after the library's construction (see The Ancient Word under Related Topics). Euclid is best known for his extensive study of geometry and number theory compiled in his multi-volume book *Elements*. Like many in the ancient world, Euclid studied multiple subjects, one of them being the study of light and optics. He documented the results of this work in a book titled *Optica*. In the book Euclid extended Empedocles' concept that we see by means of light emitted from our eyes, which became known as the emission theory. Initially he questioned this theory by posing a problem. If you close your eyes at night and then open them you see the stars immediately. If light is emitted from our eyes, reflected from an object, and then back into our eyes, how can light travel from our eyes to the distant stars and back again instantly? The answer had to be that the speed of light is infinite. Euclid also postulated that light travels in straight lines and described the phenomena of reflection from polished metal mirrors.

Around 60 AD Heron of Alexandria (~ 10 – 75 AD) expanded upon Euclid's theory of light traveling in straight lines. Heron showed by geometrical methods that light reflected from a flat polished metal mirror takes the shortest possible path, i.e., a straight line from the light source to the mirror and finally to the observer. Using this result, he concluded that the angle of reflection from a mirror must equal the angle of incidence on the grounds that this would yield the shortest path. In addition, Heron argued that the speed of light is infinite. He based his argument on the free fall of objects. An object thrown horizontally at a relatively slow speed does not travel in a straight line. Instead, gravity bends the object back toward the ground. An object thrown at a higher speed appears at first to travel in a straight line, but it too eventually curves back to Earth. Hero reasoned that if an object were thrown at an infinite speed it would travel in a straight-line forever. Since light travels in straight lines, its speed must be infinite.

Around 1,000 AD, the Muslim mathematician, astronomer, and physician al-Haytham, (also known as Alhazen) made considerable strides in the understanding of light. al-Haytham believed that light must travel at a large but finite velocity arguing that "light is substantial matter, the propagation of which requires time, even if this is hidden from our senses". He maintained that the speed of light was variable, traveling at a slower speed through denser material.

Danish astronomer Tycho Brahe (1546 – 1601) was the greatest observational genius in the age before the telescope. He built huge mechanical devices to accurately determine the positions of the planets and stars as they moved through the night sky. His observations were carefully recorded producing the most complete and accurate astronomical data of the time. Brahe was the first astronomer to describe the supernova which occurred in 1572 in the constellation Cassiopeia. According to Brahe's observations, the "new star" suddenly appeared in the sky, slowly intensified in brightness, and then faded from view over an 18-month period. The supernova caused Brahe and others to begin questioning the age-old belief in a perfect unchanging universe in which the speed of light is infinite.

Brahe's super accurate observations were the basis for Johannes Kepler's (1571 – 1630) theories of planetary motion. In addition, Kepler was the first person, after the time of al-Haytham, to make significant steps forward in the understanding of light and optics. At the beginning of the 17th century Kepler gave the first correct explanation of how the human eye works. He correctly explained short and long sightedness. He discovered that the intensity of light varies inversely with the square of the distance from the light source. However, Kepler also believed that the speed of light was infinite. Kepler concluded that if the velocity of light was finite the Sun, Moon, and Earth would be out of alignment during lunar eclipses. Since they were not, the speed of light had to be infinite.

In addition, French mathematician and scientist Rene Descartes (1596 – 1650), known for his discovery of the sine law of refraction, believed that the speed of light was infinite or instantaneous.

Galileo was the first person to attempt to measure the speed of light. In his book "Discourses and mathematical demonstrations concerning the two new sciences", published in 1638, Galileo addressed the speed of light, writing: "Everyday experience shows that the propagation of light is instantaneous; for when we see a piece of artillery fired at great distance, the flash reaches our eyes without lapse of time; but the sound reaches the ear only after a noticeable interval." In his book Galileo proposed an experiment to determine the speed of light which he later performed. Galileo and an assistant stood on separate hills a mile apart. Each had a lantern equipped with a shutter. Galileo opened the shutter on his lantern. His assistant opened the shutter on his own lantern upon seeing the light from Galileo's lantern. Galileo attempted to measure the time from opening the shutter on his lantern until seeing the light from his assistant's lantern. How Galileo measured the time interval is unclear. When Galileo performed his experiment mechanical clocks were large and not very accurate (see the chapter Measuring Time under Related Topics). It has been suggested that perhaps Galileo used a water clock or even his own pulse to measure the time interval. Whatever method he used, he was not able to perceive a difference in time between opening the shutter on his lantern and seeing the light from the lantern of his assistant. He concluded that light "if not instantaneous, is extraordinarily rapid".

3.2 Technological Breakthrough

Invention of the pendulum clock by Christiaan Huygens in 1657 provided the technological breakthrough necessary for measuring the speed of light.

Galileo had actually begun studying pendulums around 1602, apparently intrigued by the swinging motion of chandeliers. He discovered that pendulums exhibit a property known as isochronism. Isochronism means that the period of a swinging pendulum is approximately independent of the amplitude or arc of its swing. This is a critical property which makes a pendulum useful in regulating the speed of a clock. Galileo also found that the period of a pendulum is proportional to the square root of its length and independent of the mass of the pendulum bob (the weight at the end of the pendulum rod). Galileo used free-swinging pendulums in simple timing applications. For example, a particular event was completed in 50 swings. The idea of using a pendulum to build a clock occurred to Galileo around 1637. Galileo began working on a pendulum clock but died before completing the project.

Huygens' pendulum clock was based in part on the work done by Galileo. The clock was built for Huygens by clockmaker Salomon Coster who completed the clock in 1657. The accuracy of Huygens' clock was profound compared to previous mechanical clocks. Time could now be accurately measured in seconds, a resolution desperately needed by astronomers to determine the distance from the Sun to Earth, the distances to the other known planets, and ultimately to derive first estimates for the speed of light.

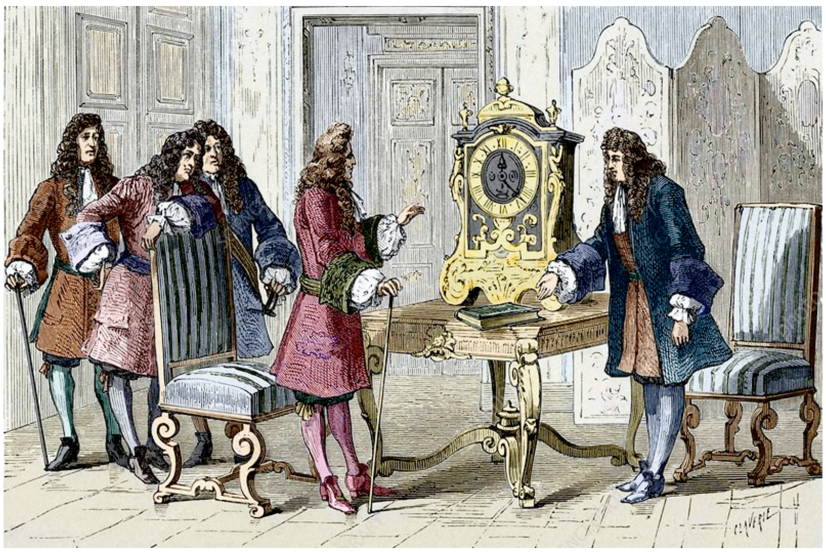

Figure 1 Huygens presenting one of his clocks to King Louis XIV of France in 1659 (source: Sciencephotolibrary)

3.3 Ole Romer (1644 – 1710)

The speed of light was first successfully measured in 1676 by Danish astronomer Ole Romer (Figure 2) while working at the Paris Observatory. He had been studying the eclipse of Jupiter with its moon Io and noticed that the beginning of an eclipse varied instead of occurring at a specific time as he expected. Over a period of months, the lag in the expected time became greater. A few months later the trend reversed. The lag between the expected and actual time of eclipse kept getting shorter. Romer concluded that the variation was due to the time required for light to cross Earth's orbit. The lag was zero when Earth was closest to Jupiter (point A in Figure 3) and longest, about 22 minutes longer, when the Earth was the furthest away from Jupiter (point B in Figure 3). From his observations Romer concluded light must take 22 minutes to traverse Earth's orbit.

Figure 2 Ole Romer (source: Wikipedia)

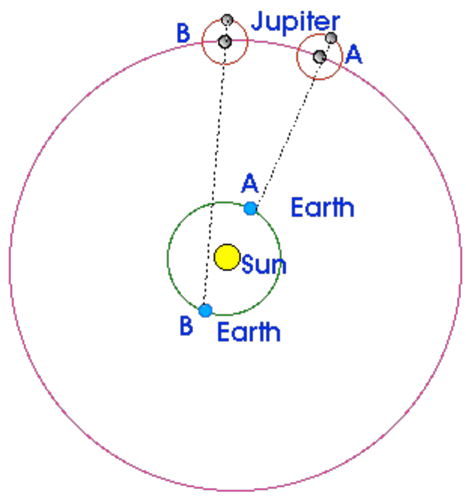

Figure 3 Romer's Experiment (source: ux1.eiu.edu)

Knowing the diameter of Earth's orbit, Romer could calculate the speed of light. However, the distance from the Sun to Earth first had to be known.

Astronomer Giovanni Cassini (1625 – 1712) and his team working at the Paris Observatory determined this distance in 1673 by measuring the parallax of Mars, that is, how far Mars shifted against the background of distant stars when viewed from two different places on Earth. This very slight shift was used by Cassini to first find the distance from Earth to Mars. With that known, the distance of Earth from the Sun could be calculated since all relative distances in the solar system had been established by

Kepler's laws and astronomical observations. Using this approach Cassini and his team determined that Earth was 139,875,342 km from the Sun. This distance is a huge number, far greater than anyone imagined. Cassini's calculation provided for the first time a clear understanding of how large the solar system actually is.

Cassini (Figure 4) was born in 1625 near Nice, Italy. In 1669 he moved to France and became a French citizen. Through a grant from Louis XIV of France, Cassini helped to build the Paris Observatory which opened in 1671. Cassini remained the director of the observatory until his death in 1712. Among his other accomplishments, Cassini discovered the four largest moons of Saturn and the division within the rings of Saturn. The division is named the Cassini Division in his honor.

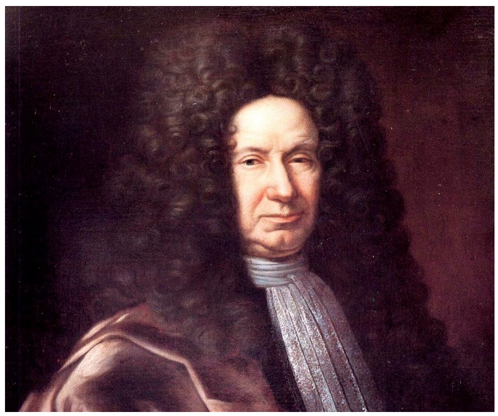

Figure 4 Astronomer Giovanni Cassini (source: Famous People)

Using the distance from the Earth to the Sun calculated by Cassini, Romer was able to determine the speed of light knowing that it took 22 minutes for light to cross Earth's orbit. The speed of light that he arrived at was 211,932 km/sec. This value is about 71% of the actual speed of light which is 299,792.458 km/sec. Two errors account for most of the difference in the speed calculated by Romer and the actual speed of light. First, the time for light to cross Earth's orbit is about 16.6 minutes instead of Rover's value of 22 minutes. Second, the distance from Earth to the Sun is 149,600,000 km instead of 139,875,342 measured by Cassini. Still, Romer's estimate for the speed of light is remarkably close to the actual value given the crude instruments available to Romer and Cassini at the time.

Based on Romer's work, Newton estimated that it took 7 to 8 minutes for light to travel from the Sun to Earth. The actual time is 8 minutes 19 seconds.

3.3.1 Determining the Distance of From Earth to the Sun (1 AU)

By the late 1600s scientists had used Kepler's three laws of planetary motion to work out the relative distances between the planets then known in the solar system. (See Heliocentric Universe under Related Topics). The distances were measured in Astronomical Units (AUs) where 1 AU is the distance from Earth to the Sun. However, they had no idea what a distance of 1 AU actually was in kilometers (or miles).

Mars is an easily observed planet. If the distance from Earth to Mars could be determine, then a distance of 1 AU could be calculated.

From Kepler's third law it was known that "The squares of the periods of the planets are proportional to the cubes of their average distances from the Sun".

In equation form

$$\frac{P_M^2}{P_E^2} = \frac{a_M^3}{a_E^3}$$

where in this case

P_M = the period of Mars = 1.88 years

P_E = the period of Earth = 1 year

a_M = the distance of Mars from the Sun

a_E = the distance of Earth from the Sun

1 AU = distance from Earth to the Sun.

We know that the orbital period of Earth (P_E) is one year and its distance from the Sun (a_E) is 1 AU. The distance of Mars (a_M) from the Sun in AUs is thus

$$a_M^3 = a_E^3 \frac{P_M^2}{P_E^2} = 1^3 \frac{(1.88)^2}{(1)^2} = (1.88)^2 = 3.53$$

$$a_M = 1.52\ AU$$

In addition, when the Sun, Earth, and Mars are in opposition, that is, in a straight line with Earth and Mars on the same side of the Sun, as illustrated in Figure 5, then

$$a_M = a_E + a_{EM}$$

$$a_{EM} = a_M - a_E = 1.52 - 1.00 = 0.52\ AU$$

where

a_{EM} = the distance from Earth to Mars.

So, if the actual distance to Mars can be determined then 1 AU = a_E is equal to

$$a_E = 1\ AU = \frac{a_{EM\ actual}}{0.52}$$

and the diameter of Earth's orbit is 2 a_E.

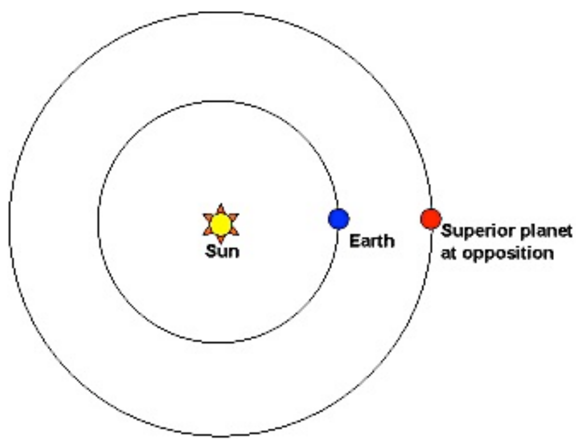

Figure 5 Astronomical Opposition (source: earthsky.org)

3.3.2 Parallax

Parallax is the difference in the apparent positions of an object when viewed along two lines of sight.

Parallax is easily demonstrated by holding a pencil out in front of you. When you alternately close one eye and then the other, the position of the pencil appears to change relative to distant objects in the background, as illustrated in Figure 6.

Since there is a distance separating your two eyes, each eye presents a different view of the pencil relative to the background. Your brain processes the two simultaneous views exploiting parallax to give you depth perception and the ability to estimate distances.

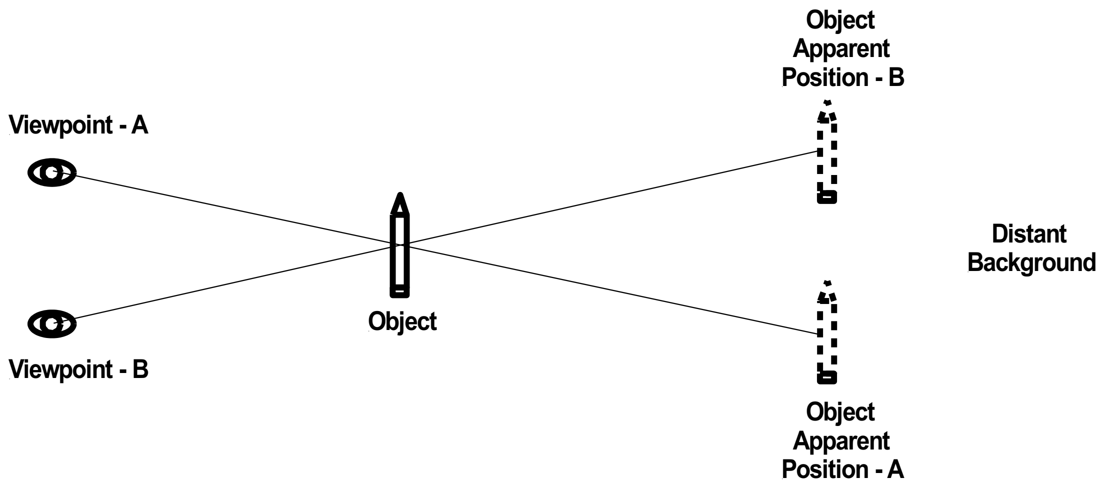

Figure 6 Example of Parallax (source: author)

Parallax is measured by the semi-angle q shown in Figure 7. The complete angle Q between the two lines of sight is twice the semi-angle q.

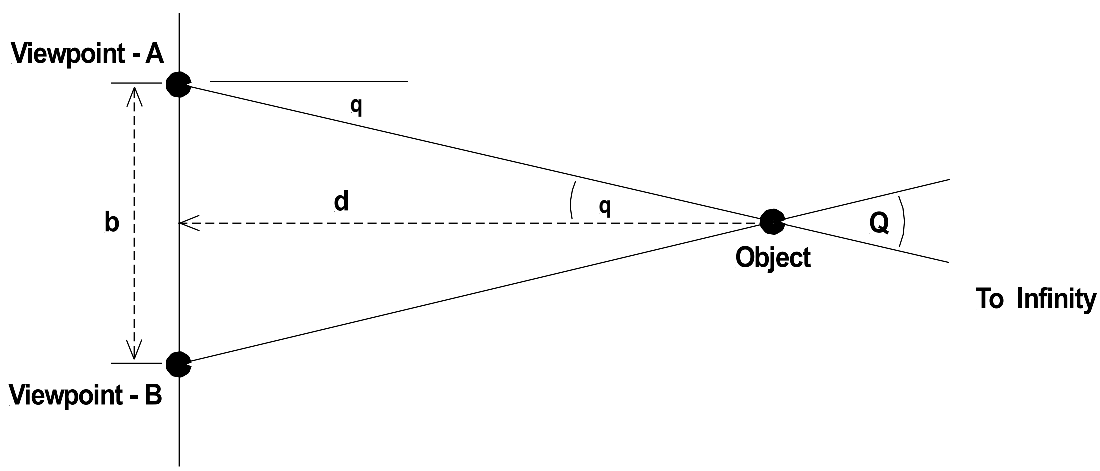

Figure 7 Measuring Parallax (source: author)

The distance b in Figure 7 is called the baseline. If the baseline and the parallax angle q are known, then the distance d to the object can be determined as follows:

$$\tan q = \frac{(b/2)}{d}$$

$$d = \frac{(b/2)}{\tan q}$$

3.3.3 Calculating the Distance From Earth to Mars

It was known that Mars and Earth would be in opposition in September and October of 1672. In addition, English astronomer John Flamsteed had predicted that Mars would pass in front of a particular star in the constellation Aquarius (the middle Psi star) on the first of October, 1672 (Figure 8).

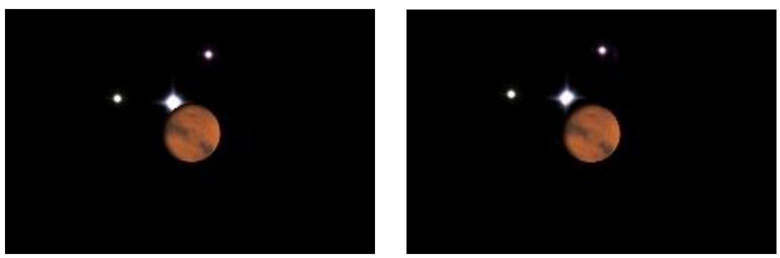

Figure 8 Mars passing in front of Aquarius (source: Christine M. Rodrigue, Ph.D.)

This prediction, coupled with opposition, created considerable excitement in the astronomical community. Cassini decided to exploit this opportunity. With the Sun, Earth and Mars in a straight line it would be possible to determine the distance from Earth to Mars (a_{EM}) and from that the actual value of 1 AU.

To obtain the distance from Earth to Mars, Cassini sent his friend and collaborator, Jean Richer, to the town of Cayenne in French Guiana on the north-east coast of South America. The great circle distance from Cayenne to Paris, measured along the surface of the Earth, is 7,089 km. However, the distance required for Cassini's calculations is actually the chord distance from Cayenne to Paris which is roughly 6,700 km, as shown in Figure 9.

Richer made precise measurements of the position of Mars with respect to the Psi star in Aquarius at midnight on a specified evening. At midnight that same evening, from the Paris Observatory, Cassini and another collaborator, Jean Picard, made the same measurement of Mars' position relative to Psi. It was nearly a year later when Richer arrived back in Paris with his data.

Using Figure 9, the angle q measured by Cassini's team was 9.5 arcsec. Converting arcsec to degrees gives:

$$q = \frac{9.5\ arcsec}{(60\ sec)(60\ min)} = 0.0026389°$$

The distance from Earth to Mars calculated by Cassini was then

$$a_{EM\ actual} = d = \frac{(b/2)}{\tan q} = \frac{(6{,}700/2\ )}{4.61 \cdot 10^{-5}} = 72{,}735{,}178\ km$$

This value differs from today's measurement by about 7%.

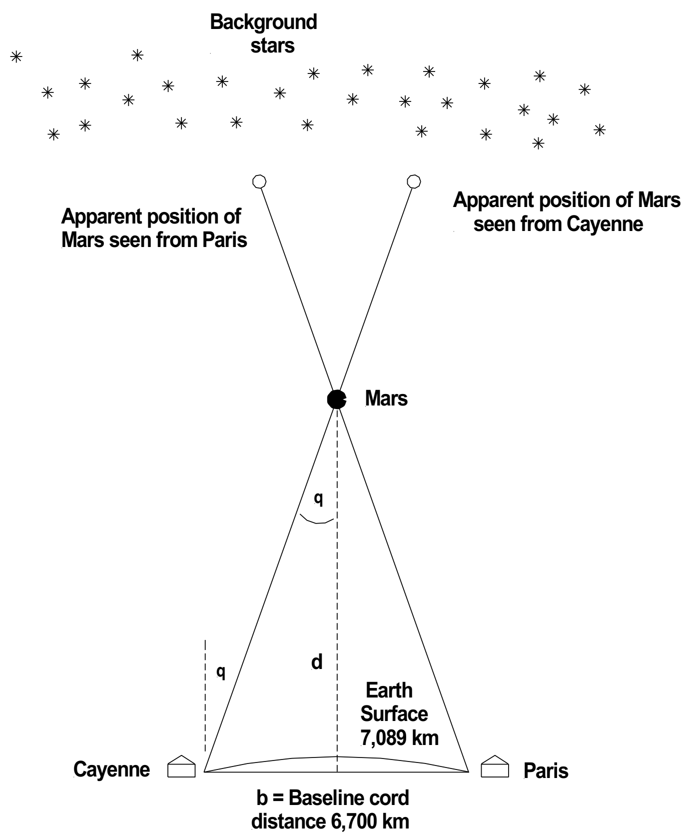

Figure 9 Parallax of Mars (source: author)

3.3.4 Calculating the Distance from Earth to the Sun (1 AU)

Knowing the distance to Mars, the distance from Earth to the Sun (one Astronomical Unit = 1 AU) can be calculated. This distance calculated by Cassini is equal to

$$a_E = 1\,AU = \frac{a_{EM\,actual}}{0.52} = \frac{72{,}735{,}178}{0.52} = 139{,}875{,}342\ km$$

3.3.5 Romer's Speed of Light Calculation

The distance from Earth to the Sun allowed Romer to calculate the speed of light (c) given that it took light 22 minutes to travers Earth's orbit. The calculation made by Romer was

$$c = \frac{2 \cdot a_E}{(22\ min)(60\ min/sec)} = 211{,}932\ km/sec$$

This value is about 71% of the actual speed of light which is 299,792.458 km/sec.

3.4 James Bradley (1692 – 1762)

Like Romer before him, Bradley (Figure 10) was not attempting to measure the speed of light. Instead, in December 1725 through the following year he was attempting to measure the distance to stars using parallax. The star Bradley selected was $\gamma - Draconis$, a second magnitude star that passes directly over-head above London. The baseline for the measurement was the diameter of Earth's orbit, Figure 11. The advantage of measuring the parallax for a star passing directly over-head is that it eliminates errors in measurement due to refraction in Earth's atmosphere.

Figure 10 James Bradley (source: onthisday)

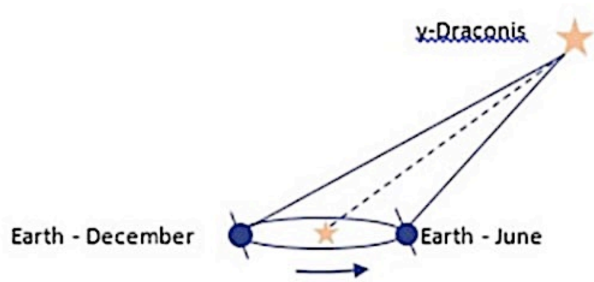

Figure 11 Measuring the parallax of $\gamma - Draconis$ (source: John Ford)

Bradley's attempt to measure the distance to $\gamma - Draconis$ was unsuccessful. $\gamma - Draconis$ is simply too far away. We know today that $\gamma - Draconis$ is 154 light-years for Earth. At that distance, the parallax angle for $\gamma - Draconis$ is about 21 milliarcseconds or about 6×10^{-6} degrees, way too small to measure with the instruments available to Bradley. The first reliable parallax measurement of a star was performed over a hundred years later by Friedrich Bessel in 1838.

Bradley ran into an unexpected problem when attempting to measure the parallax of $\gamma - Draconis$. From December to March the star moved progressively south from its true position reaching a maximum deviation of approximately 20" (20 arc seconds) in March (Figure 12). From March to June, it moved north reaching zero deviation in June. From June to September is continued moving northward until it was 20" north of its true position in September. Finally, from September to December it moved south again reaching zero deviation in December, as illustrated in Figure 12.

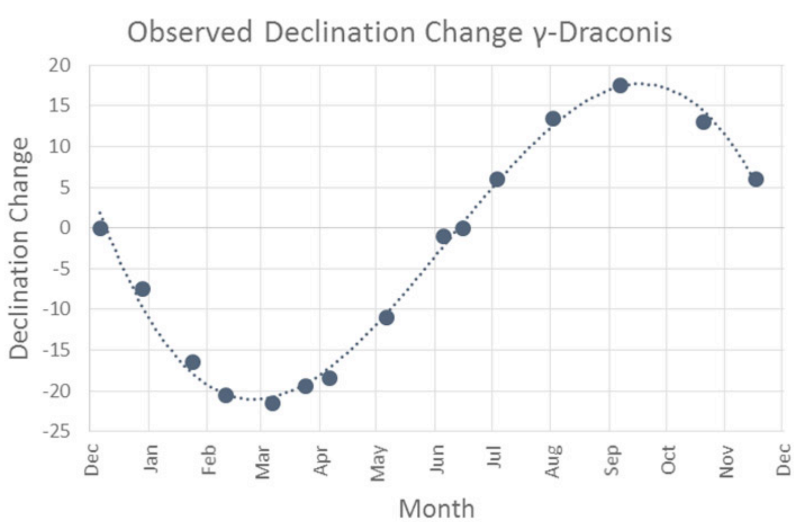

Figure 12 Changes in the observed position of $\gamma - Draconis$ (source: John Ford)

After considerable work (verifying that his telescope was working properly, making repeated observations, ruling out the wobble of Earth on its axis of rotation, etc.) he concluded that the motion of the Earth in its orbit caused the position of $\gamma - $ Draconis to vary slightly ($\pm$ 20") though the year. He called this phenomenon the aberration of light.

The classic explanation of aberration is illustrated in Figure 13.

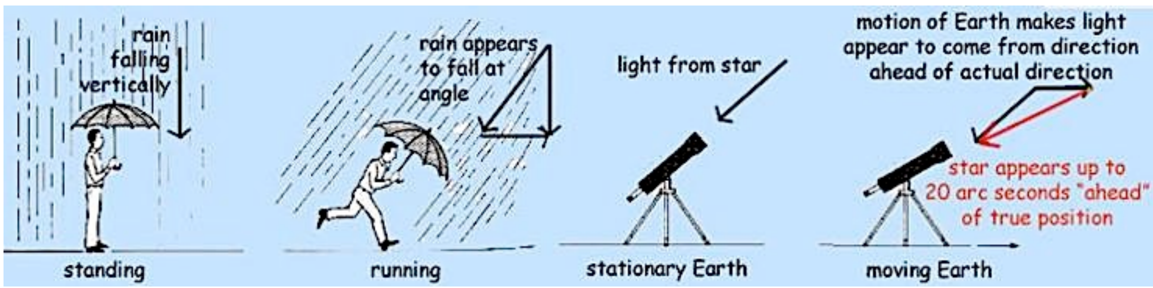

Figure 13 Explanation of aberration (source: astronomy.stackexchange.com)

To stay dry, we hold an umbrella directly over our head when standing in rain that is falling straight down (no wind). However, if we start to run the rain appears to fall at an angle toward us as illustrated in Figure 13. To stay dry, we have to hold the umbrella downward in front of us.

The same is true when viewing a star. The star appears in its true position when Earth is stationary with respect to the star. However, as the Earth moves in its orbit, the position of the star changes, appearing ahead of its true position as illustrated in Figure 14. In December and June the apparent position of the star is close to its true direction. However, in March and September a noticeable angle exists between the star's true and apparent location. The angle is in the direction of the Earth's motion around the Sun, that is, the star appears ahead of where it should be.

The angle between the star's true and apparent position depends on the velocity of the Earth with respect to the star and the speed of light. Knowing this angle and the speed of Earth in its orbit around the Sun allows the speed of light to be determined. Bradley showed that the maximum angle θ by which a star is displaced from its mean position is

$$\theta = v/c$$

where

v = the speed of Earth in its orbit around the Sun

c = the speed of light.

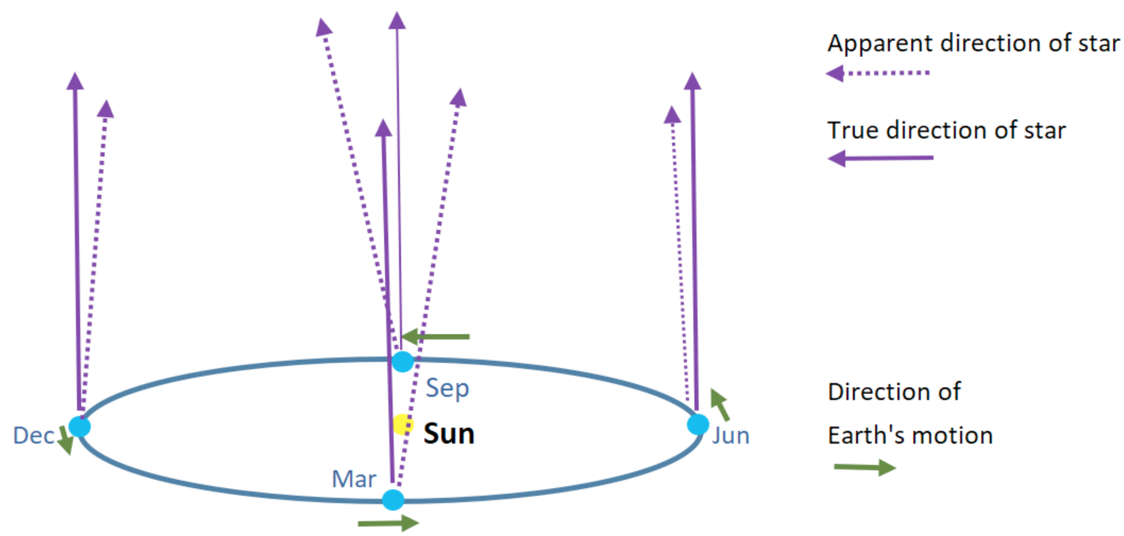

Figure 14 Stellar Aberration (source: Hurley)

Bradley determined this angle to be 20.5 arc seconds (20.5”). There are 60 arc seconds per arc minute and 60 arc minutes per degree. Thus

$$1° = 3{,}600 \; arc \; seconds$$

Also, there are 2π radians in a circle ($2\pi \, rad = 360°$). Thus

$$1 \, rad = \frac{180°}{\pi} = 57.2958°$$

The maximum aberration determined by Bradley of 20 arc seconds is thus equal to

$$\theta = 20 \; arcsec = \frac{20 \, \pi}{3600 \cdot 180} = 9.696 \; x \; 10^{-5} \; rad$$

During the time of Bradley it was believed that Earth traveled around the Sun at a speed of 29.2 km/s. Today we know that Earth’s average speed in its elliptical orbit is 29.8 km/s. Using Bradley’s orbital speed for Earth

$$c = \frac{v}{\theta} = \frac{29.2 \; km/s}{9.696 \; x \; 10^{-5} \; rad} = 301{,}155 \; km/s$$

which is within 0.5% of the correct value (c = 299,792.458 km/s).

Derivation of the equation c = v/θ , as explained by Steve Hurley in his paper "Stellar Aberration", is provided below.

Figure 15 shows a ray of light from a star as seen with respect to the Sun (in the Sun's reference frame). The x-axis is the direction of Earth's motion in its orbit around the Sun. The y-axis is perpendicular to Earth's orbit. The ray of star light, traveling at the speed of light c, makes an angle ϕ_S to the plane of Earth's orbit as seen relative to the Sun.

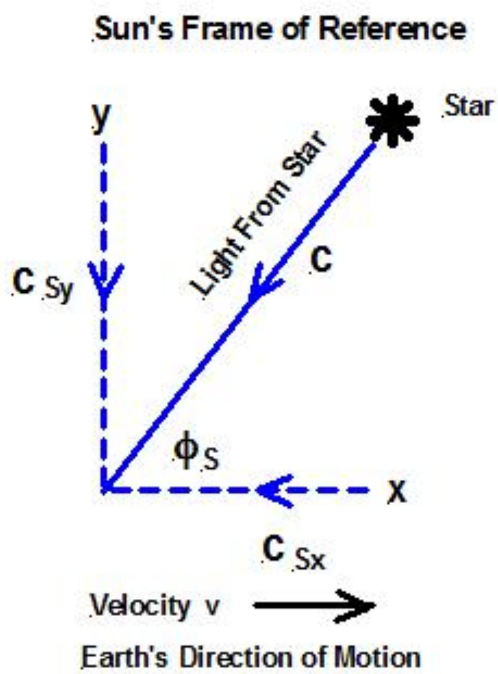

Figure 15 Aberration – Sun's frame of reference (source: Hurley)

The velocity of light from the star can be decomposed into orthogonal x and y components as shown in Figure 15.

- c_{Sx} is the velocity of star light in the direction of Earth's motion
- c_{Sy} is its velocity perpendicular to Earth's motion

In equation form

$$c_{Sx} = c \cdot \cos \phi_S$$

$$c_{Sy} = c \cdot \sin \phi_S$$

The tangent of ϕ is

$$\tan\phi = \frac{c_{Sy}}{c_{Sx}} = \frac{\sin\phi_S}{\cos\phi_S}$$

In Earth's reference frame Earth is moving at a velocity v in the x direction relative to the Sun as shown in Figure 16.

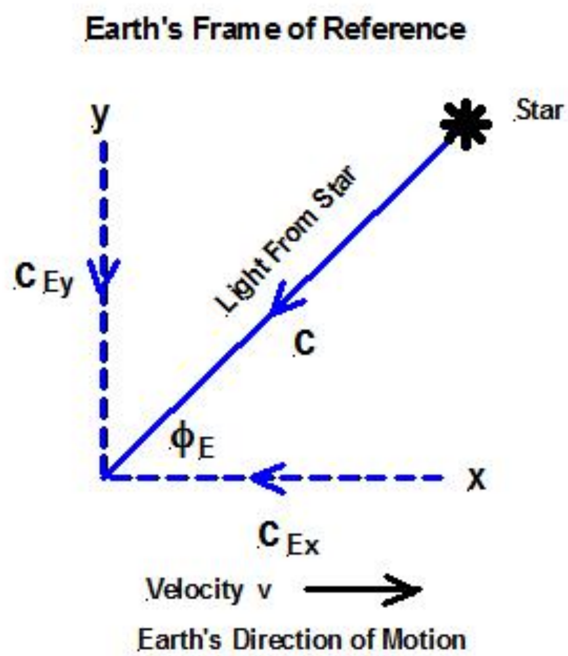

Figure 16 Aberration – Earth's frame of reference (source: Hurley)

From Earth the star appears at an angle ϕ_E with respect to Earth's motion. In Earth's reference frame the x and y components of the star light velocity are

- c_{Ex} is the velocity of star light in the direction of Earth's motion (Earth reference)
- c_{Ey} is its velocity perpendicular to Earth's motion (Earth reference).

The y component of the star light velocity is the same in both reference systems. Thus

$$c_{Ey} = c_{Sy}$$

The x component of the star light velocity in Earth's frame of reference is

$$c_{Ex} = c_{Sx} + v$$

where v is the velocity of Earth in its orbit.

$$c_{Ey} = c_{Sy} = c \cdot \sin \phi_S$$

$$c_{Ex} = c_{Sx} + v = c \cdot \left[(\cos \phi_S) + \frac{v}{c} \right]$$

The tangent of ϕ_E is

$$\tan \phi_E = \frac{c_{Ey}}{c_{Ex}} = \frac{c_{Sy}}{c_{Sx} + v} = \frac{c \cdot \sin \phi_S}{c \cdot [(\cos \phi_s) + v/c]} = \frac{\sin \phi_S}{[(\cos \phi_s) + v/c]}$$

$$\tan \phi_E = \frac{\sin \phi_S}{[(\cos \phi_s) + v/c]}$$

The angular displacement θ of the star due to Earth's movement

$$\theta = \phi_E - \phi_S$$

The tangent of the difference of two angles is

$$\tan \theta = \tan(\phi_E - \phi_S) = \frac{\tan \phi_E - \tan \phi_S}{1 + \tan \phi_E \cdot \tan \phi_S}$$

$$\tan \theta = \frac{\dfrac{\sin \phi_S}{\cos \phi_S + v/c} - \dfrac{\sin \phi_S}{\cos \phi_S}}{1 + \dfrac{\sin \phi_S}{\cos \phi_S + v/c} \cdot \dfrac{\sin \phi_S}{\cos \phi_S}}$$

$$\tan \theta = \frac{-(v/c) \sin \phi_S}{(\cos \phi_S)^2 + (v/c) \cos \phi_S + (\sin \phi_S)^2}$$

since

$$(\sin \phi_S)^2 + (\cos \phi_S)^2 = 1$$

$$\tan \theta = \frac{-(v/c) \sin \phi_S}{1 + (v/c) \cos \phi_S}$$

The velocity of the Earth v is much smaller than the velocity of light c. That is

$$v/c \ll 1$$

consequently

$$\tan\theta = -(v/c)\sin\phi_S$$

If $\phi_S = 90°$ (the star is directly over head as in Bradley's experiment), then

$$\sin\phi_S = 1 \quad and \quad \tan\theta = -v/c$$

The difference between ϕ_S and ϕ_E is very small so

$$\theta = \phi_E - \phi_S$$

is also very small. Consequently, to a good approximation

$$\theta = \tan\theta$$

and

$$\theta = -v/c$$

which is the answer that we were looking for.

The aberration angle θ in arc seconds as a function of ϕ_E (the angle of a star with respect to Earth's motion) is given by

$$\theta = 3600 \cdot \tan^{-1}[(v/c)\sin\phi_E]$$

and shown in the following table for various values of ϕ_E.

Angle ϕ_E of star relative to Earth's motion (degrees)	Aberration angle θ (arcsec)
90	20.5
75	19.8
60	17.8
45	14.5
30	10.3
15	5.3
0	0

The aberration angle of a star is independent of the star's distance from Earth. The parallax angle of a star is not. The parallax angle for a star becomes smaller the further away a star is. For example, the parallax angle for the star Proxima Centauri, the closest star to the Sun, is only 0.769 arcsec. The parallax angle of all other stars is smaller, usually much smaller. So, it is no wonder that Bradley could detect and measure aberration angles but not parallax.

An interesting situation is shown in Figure 17. In this figure the star being observed is in the same plane as Earth's orbit.

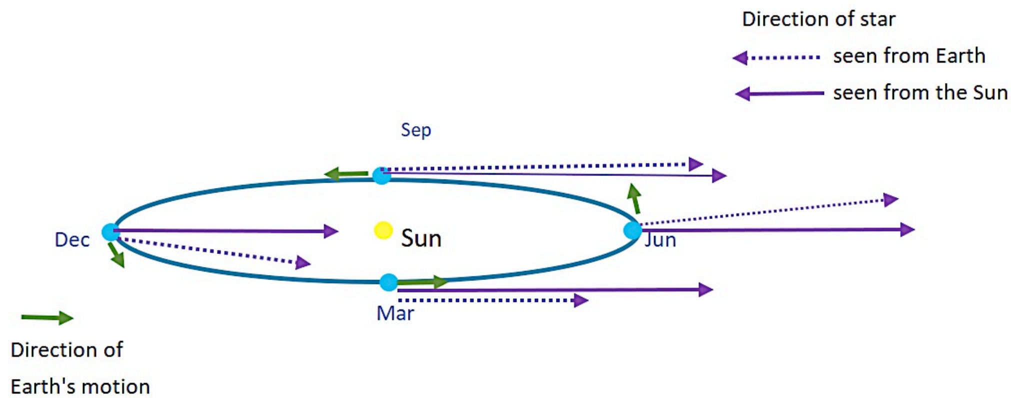

Figure 17 Star lies in Earth's extended orbital plane (source: Hurley)

In March the Earth is moving directly toward the star. So, the angle ϕ_E between the star and Earth's direction of motion is zero. Consequently, the star's aberration angle $\theta = 0$ according to the above table. That is, the star is not displaced, it appears in its true position.

In June the Earth is moving 90 degrees to the direction of the star ($\phi_E = 90°$) so the star is displaced 20.5 arcsec in the direction of Earth's movement, its maximum displacement.

In September the Earth is moving directly away from the star so the angle between the star and Earth's direction of motion is $\phi_E = 180$. Since $\sin 180 = 0$ the star's aberration angle is $\theta = 0$. The star is not displaced from its true position.

In December the Earth is again moving at 90 degrees to the direction of the star (but in the opposite direction from June). As a result, the star is displaced 20.5 arcsec in the direction of Earth's movement as illustrated in Figure 17.

3.5 Armand Fizeau (1819 – 1896)

The first non-astronomical determination of the speed of light was performed by French scientist Armand-Hippolyte-Louis Fizeau in 1849 (Figure 18). Fizeau greatly improved on Galileo's failed attempt to measure the speed of light. Instead of uncovering lanterns, as Galileo did, Fizeau used a rotating toothed wheel, mirrors, lenses, and a light source. His set-up is shown in Figure 19. A lens was used to focus a light source onto a partially reflective mirror positioned at an angle of 45°. Some of the light passed downward (in the figure) through the mirror. The remainder of the light was reflected by the mirror and passed through the gap between two teeth of the wheel. This light was reflected back by a mirror placed on a hill top along the banks of the Seine River. This mirror was 8.633 km from the home of Fizeau 's parents in Suresnes, just west of Paris, where he conducted his experiment.

Figure 18 Armand-Hippolyte-Louis Fizeau (source: Wikipedia)

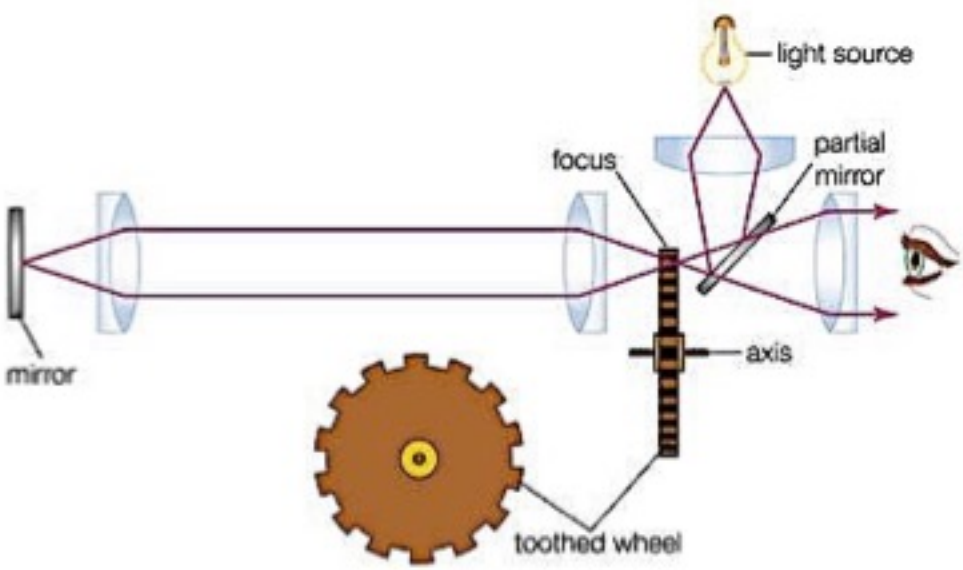

Figure 19 Fizeau's toothed wheel experiment (source: Encyclopedia Britannica)

If the wheel was stationary, light reflected from the distant mirror would pass through the gap between the same pair of teeth as the outgoing light. The reflected light would then continue through the 45° partial mirror to the observer. Because of the partial mirror, the only light that the observer could see was that reflected from the distant mirror. Since the wheel was stationary, the observer would see a continuous reflection of light.

Rotating the wheel at a high speed "chopped" the outgoing light beam into pulses as teeth of the wheel periodically blocked the outgoing light (covering and uncovering the lantern as Galileo tried to do). If the wheel were rotated fast enough a tooth would move into position blocking the reflected pulse. In this case the observer would not see any reflected light from the distant mirror. This was the desired result. The speed of light could be calculated knowing the distance to the remote mirror, the number of teeth on the wheel, and the speed at which it was rotating. The calculations are as follows.

The speed of light c is

$$c = \frac{d}{t}\ meters\ per\ second\ (m/s)$$

where d is the distance that the light travels in t seconds. If L is the distance from the toothed wheel to the remote mirror, then the total distance traveled by the light from the wheel to the remote mirror and back again, is d = 2L.

The time required to traverse this distance is then

$$t_{GT} = \frac{d}{c} = \frac{2L}{c}$$

In this time frame the next tooth must move into position to block the pulse of light reflected from the distant mirror.

If the wheel is rotating at f revolutions per second and n is the number of teeth on the wheel, then teeth are passing in front of the observer at a rate of

$$n \cdot f \qquad teeth\ per\ second$$

The time t_T is the time for the wheel to rotate one tooth position. This time is simply

$$t_T = \frac{1}{n \cdot f} \quad seconds\ per\ tooth$$

However, the time that we are interested in is t_{GT}, the time for light to travel to the distant mirror and back. The time it takes the wheel to rotate one half tooth position must be equal to t_{GT} . Light shines through the gap between two teeth at the beginning of this time period. At the end of the time period the next tooth has rotated into position so that it is blocking propagation of light through the wheel. Both out going and reflected light are blocked. Thus

$$t_{GT} = \frac{1}{2}t_T = \frac{1}{2 \cdot n \cdot f}$$

and speed of light is

$$c = \frac{2L}{t_{GT}} = (2L)(2nf) = 4nfL$$

In Fizeau's experiment the wheel had 720 teeth (n = 720) and rotated at 756 rotations per minute or $f = 12.6$ rotations per second. The distance L was equal to 8.633 km. Thus, the speed of light calculated by Fizeau was

$$c = 4nfL = 4(720)(12.6)(8.633) = 313{,}274.304\ km/s$$

The speed that Fizeau calculated was roughly 5% higher than the actual speed of 299, 792.458 km/s.

3.6 Jean-Bernard-Leon Foucault (1819 – 1868)

English scientist and inventor Charles Wheatstone is best known for development of the Wheatstone bridge used to measure unknown electrical resistances. He was also a major player in development of the telegraphy. In 1834 Wheatstone developed a method of using a rapidly rotating (spinning) mirror to study transient phenomena, including the duration of an electric spark. Wheatstone suggested to French mathematician, physicist and astronomer Francois Arago that a spinning mirror could possibly be used to study the speed of light. In an 1838 publication Arago expanded on Wheatstone's ideas suggesting that a spinning mirror could be used to, at least partially, resolve the particle verse wave theories of light by determining the relative speed of light in air and water.

In 1845 Arago recommended that Fizeau and Foucault (Figure 20) attempt to measure the speed of light using the rotating mirror concept. Fizeau and Foucault were initially friends and collaborators working together taking images of the Sun and characterizing absorption bands in the infrared spectrum of sunlight. However, in 1849 they apparently had a falling out and went their separate ways. In 1849 Fizeau determined the speed of light in air using his toothed wheel apparatus described in the previous section. In 1850 Foucault and Fizeau both used rotating mirror devices to determine the relative speed of light in air versus water.

In his experiment, Foucault placed a tube of water in the path between a spinning mirror and a distant fixed mirror as illustrated in Figure 21.

Figure 20 Jean-Bernard-Leon Foucault (source: Wikipedia)

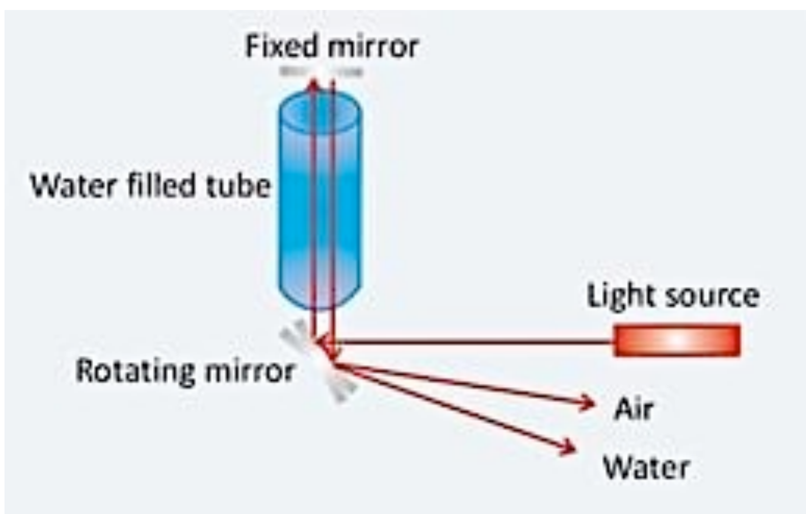

Figure 21 Foucault's air vs water speed of light experiment
(Source: University of Virginia http://galileo.phys.virginia.edu)

Initially the tube was empty. Light reflected by the spinning mirror passed through the empty tube and was reflected by the fixed mirror at the tube's opposite end. The reflected light from the fixed mirror traveled back through the tube to the spinning mirror.

However, the spinning mirror had only rotated slightly in the time taken for the light to pass through the tube and back again. Consequently, the returning beam of light was displaced slightly from the out-going light beam. The returning beam of light is labeled “Air” in Figure 21. The tube was then filled with water and the experiment repeated. This time the returning beam (labeled “Water” in Figure 21) was displaced more than when the tube was empty. That is, the mirror rotated further in the time for the light to pass through the tube of water and back again indicating that the speed of light through water was slower than that through air.

Fizeau performed a similar experiment. However, Foucault announced his results first. Fizeau’s results confirmed those of Foucault.

In the late 1600s Newton proposed his particle (corpuscle) theory of light. (See Newton under Related Topics). Newton predicted that light would travel faster through a more dense material. In contrast, around the same time, Christiaan Huygens and Robert Hooke proposed a wave theory of light predicting that light would travel slower through a dense material. The experiments of Foucault and Fizeau, nearly two hundred years later, showed that Huygens and Hooke were correct, putting an end to the long accepted particle theory of Newton.

In 1862, Foucault used a scaled-up version of the spinning mirror apparatus to perform an absolute measurement of the speed of light. This apparatus is shown in Figure 22.

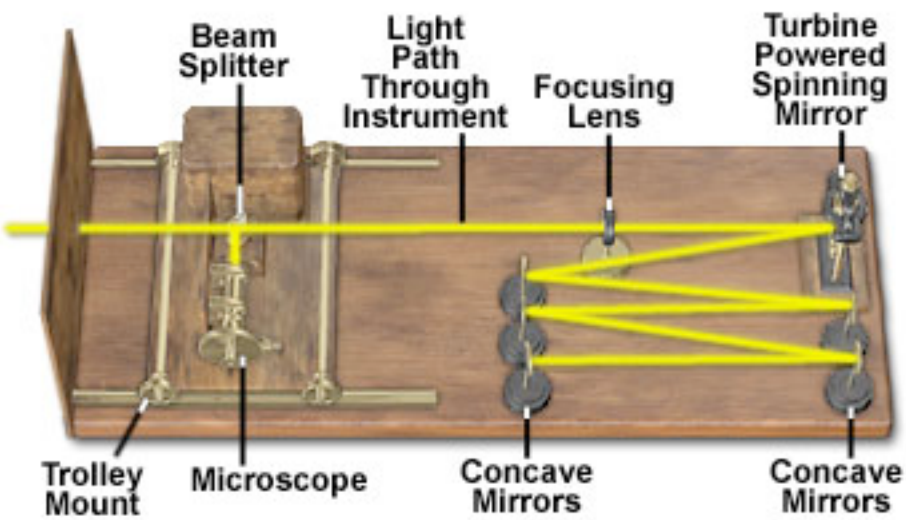

Figure 22 Foucault’s spinning mirror device (source: Olympus - Speed of Light)

In this experiment light from a source passed through a small aperture in the wall on the left side of the apparatus. The aperture formed a light beam that impinged on a beam splitter (a partially silvered mirror at an angle of 45° to the path of the beam). Part of the beam was diverted into a microscope while the rest of the beam continued on to a rapidly spinning mirror driven by a compressed air turbine. Light reflected from the spinning mirror was in turn reflected in a zig-zag manner through a series of stationary mirrors and back again to the spinning mirror. The spinning mirror turned slightly in the time that the light traveled through the series of fixed mirrors and back. Consequently, the path followed from the spinning mirror back to the beam splitter and then to the microscope was slightly displaced from that of the out-going beam. The displacement was very small but could be measured with the microscope.

If the distance traveled by the light from the spinning mirror through the series of fixed mirrors is L meters, then the distance through the mirrors and back again to the spinning mirror is 2L. The time t that it takes light to travel this total distance is

$$t = \frac{2L}{c}$$

where c is the speed of light in meters per second (m/s). If ω is the angular speed of the spinning mirror in radians per second, then the angle that the mirror turns in t seconds is

$$\theta = \frac{2L\omega}{c} = \omega t$$

This angle was viewed in the microscope as the displacement d between the out-going and returning beams of light. This displacement was less than 1 mm, but could be accurately measured by means of the microscope. The tangent of θ is

$$\tan\theta = \frac{d}{l}$$

where l is the distance from the spinning mirror through the beam splitter to the microscope. By measuring d and knowing l, Foucault was able to determine the angle θ. The speed of light c was then

$$c = \frac{2L\omega}{\theta}$$

The speed of light calculated by Foucault was

$$c = 298{,}000 \text{ km/s}$$

This value was within 0.6% of the modern value for the speed of light which is 299, 792.458 km/s.

3.7 James Clerk Maxwell (1831 – 1879)

Initially it was believed that electricity and magnetism were completely separate entities. However, from the early through mid 1800s it slowly became evident that electricity and magnetism were in fact strongly related. In 1865 Scottish physicist and mathematician James Maxwell, Figure 23, published a set of 20 equations that unified electricity and magnetism into a single theory of electromagnetics. Later Oliver Heaviside, using vector calculus (see Vector Analysis under Related Topics), simplified Maxwell's equations into the set of 4 equations that we use today.

Maxwell is regarded as one of the greatest theoretical physicist of the 19th century. Maxwell died young at the age of 48 from abdominal cancer. However, in this short life, he formulated the unifying theory of electromagnetics, developed the kinetic theory of gases, made significant contributions to the understanding of color vision, and studied the nature of Saturn's rings.

Figure 23 James Clerk Maxwell (source: University of St Andrews)

In developing his set of equations, Maxwell integrated together laws of electricity, magnetism, and induction originally developed by Gauss, Faraday, and Ampere. Maxwell's equations showed that it was possible for time varying electric and magnetic fields to be self-sustaining, each inducing the other. That is, a time varying magnetic field could induce a time varying electric field which in turn induced the magnetic field. Based on this result, Maxwell predicted the existence of electromagnetic waves, each wave consisting of an alternating electric wave perpendicular to an inseparable magnetic wave, with both orthogonal to the direction of wave travel as illustrated in Figure 24. Since each wave induced the other, the electromagnetic wave itself could theoretically propagate forever in its direction of travel through empty space.

Maxwell noticed that the inverse square root of electric permittivity in free space, ε_0, and free space magnetic permeability, μ_0, had the units of velocity. Maxwell concluded that the velocity, v, of his proposed electromagnetic wave must be

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Coulomb and Ampere had previously determined the values of ε_0 and μ_0 through careful static electric and magnetic measurements. Using their measurements Maxwell calculated the speed of an electromagnetic wave to be 284,000,000 meters per second, a value very close to the speed of light known at the time. Maxwell concluded that light itself must be an electromagnetic wave.

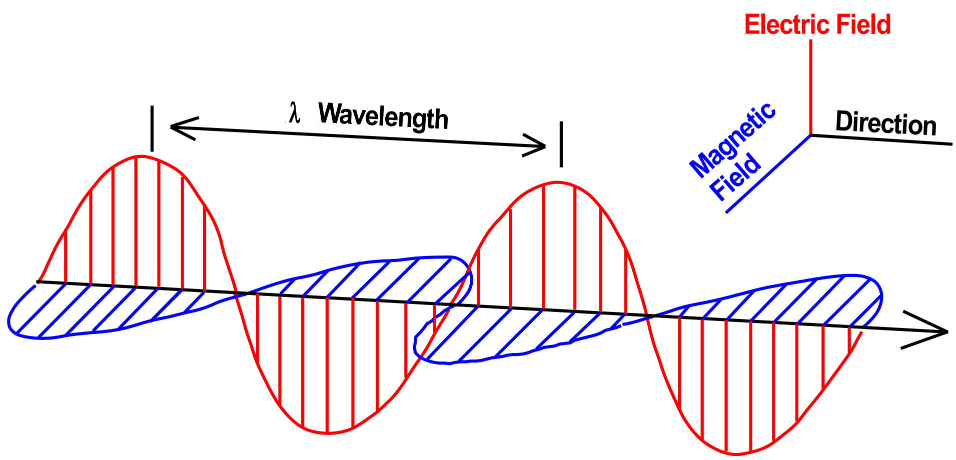

Figure 24 Electromagnetic Wave (source: author)

The next chapter describes Maxwell’s equations in more detail.

3.8 Albert A. Michelson

Albert Michelson, Figure 25, spent a good portion of his life studying the speed of light and related topics. His measurements for the speed of light, which he continued to refine throughout his life, were the most accurate available, gradually approaching the value of 299, 792.458 km/s that we accept as the speed of light today.

Michelson was born on December 19, 1852 in Strelno, Prussia which is now Strzelno, Poland. He died on May 9, 1931 in Pasadena, California after a brilliant career which included receiving the Nobel Prize for Physics in 1907.

Michelson migrated to the United States with his parents when he was two years old. His family initially settled in the silver mining town of Virginia City, Nevada before finally moving to San Francisco where his father became a prosperous merchant.

Figure 25 Albert Michelson (source: Wikipedia)

Upon graduating from high school, Michelson was appointed by President Grant to the U.S. Naval Academy in Annapolis. He graduated from Annapolis in 1873 with the rank of Ensign and began a two year cruise in the West Indies. In 1875 he returned to Annapolis becoming an instructor in physics and chemistry at the Academy. While at the Academy, Michelson began his work on measuring the speed of light. Using private funding and Naval Academy facilities he measured the speed of light to be 299,910 km/sec in January 1879.

In that same year Simon Newcomb (1835 – 1909) received a grant of $5,000 to conduct experiments to determine the speed of light. Michelson was assigned to the U.S. Naval Observatory (USNO) in Washington D. C. to assist Newcomb with his measurements.

The instrument designed by Newcomb consisted of a four-sided steel rotating mirror mechanism. The four-sided mirror was rotated at a rate up to 250 revolutions per second using compressed air directed on a fan wheel assembly. The speed of rotation could be gradually altered by adjusting the flow of the air jets. Steel mirrors were used instead of glass to prevent the mirrors from fracturing due to centrifugal force at the high rotational speed of the mirrors.

The experiments were conducted at Ft. Meyers Army base in Arlington, VA just across the Potomac River from Washington D. C. Fixed mirrors for the experiment were located at the U.S. Naval Observatory, in the Foggy Bottom district of Washington D C., and the

Washington Monument. The distances from the rotating mirror to the two fixed mirror locations was determined by the U.S. Coast and Geodetic Survey Agency.

Michelson participated in Newcomb's experiment until September 1880 when he was granted a leave of absence by the Navy to study optics in Europe. Michelson believed he had to be better educated in the field of optics if he were going to make further measurements of the speed of light. Michelson spent two years in Europe studying in Berlin, Heidelberg, and Paris.

Newcomb continued his experiments through 1882. In his final report he presented two values for the speed of light. The first value of 299,860 km/sec was determined by observations that he had made in 1882 which he believed were less affected by systematic errors. The second value of 299,810 km/sec was based on the weighted average of observations made through the course of the experiment. It turned out that his second value was actually more correct. In his reported he described changes to his equipment that he felt would lead to more accurate results. He also pointed out that conducting the experiment in either the Rocky Mountains or the California Sierra Nevada range would allow the fixed mirrors to be placed in clear view as much as 50 km away from the rotating mirror.

3.9 Michelson and Morley Experiment

At the time it was believed by most physicists that light waves, including those from distant stars, traveled through a luminiferous (light-bearing) ether that existed throughout all of space. This was a reasonable assumption since all known waves at the time flowed through some type of medium, for example ocean waves through water, sound waves through air, etc. However, the luminiferous ether had to have very unusual characteristics. While it pervaded all of space, it did not seem to have any affect on planetary motion as the planets orbited the Sun. In fact, it did not seem to affect any known physical phenomena. It apparently was very "thin" providing a medium in which light could travel without affecting anything else.

While in Europe Michelson became interested in measuring Earth's velocity through the ether. Michelson reasoned that Earth's motion would create an "ether wind" that would be observable on Earth's surface as illustrated in Figure 26. Light traveling against the wind would be slowed down while the speed of light traveling perpendicular to the wind would be unaffected.

For example, suppose that a boat is traveling in a river. The boat travels at a speed of v_B upstream a distance of d against the river current which has a speed of v_R. The boat then turns around and travels back down the river the same distance d but this time traveling with the river current. What is the total time for the boat to make the round trip to point d and back?

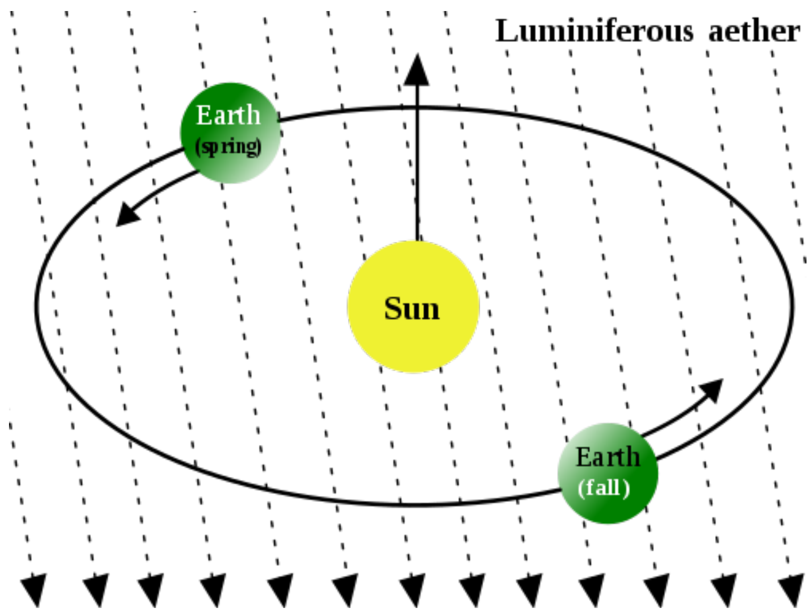

Figure 26 Earth traveling through luminiferous ether (source: Wikipedia)

The time t to travel a distance d at a speed v is

$$t = \frac{d}{v}$$

So, the time to travel a distance d up stream against the river current is

$$t_u = \frac{d}{v_B - v_R}$$

The time to travel the same distance d back down the river with the river current is

$$t_b = \frac{d}{v_B + v_R}$$

Putting these two together, the total time to travel up river and back again is

$$t_T = t_u + t_b = \frac{d}{v_B - v_R} + \frac{d}{v_B + v_R} = \frac{d[(v_B + v_R) + (v_B - v_R)]}{v_B^2 - v_R^2}$$

which after simplification becomes

$$t_T = \frac{2dv_B}{v_B^2 - v_R^2}$$

As an illustration, suppose $v_B = 6\,mph$ and $v_R = 3\,mph$, then the total time to go up the river a distance of d miles and back again is

$$t_{TR} = \frac{2dv_B}{v_B^2 - v_R^2} = \frac{2d(6)}{(6)^2 - (3)^2} = \frac{12d}{27} = 0.444 \cdot d \quad hours$$

Now suppose that the boat is in a lake instead of a river. How long does it take the boat to travel d miles up the lake and back again? In this case $v_B = 6\,mph$ as before but $v_R = 0$ because there is no current in the lake. The total time to go up the lake and back again is

$$t_{TL} = \frac{2dv_B}{v_B^2 - v_R^2} = \frac{2dv_B}{v_B^2} = \frac{2d}{v_B} = \frac{2d}{6} = 0.333 \cdot d \quad hours$$

It clearly takes more time to travel a distance d up a river and back again than it takes to go the same distance on a lake.

This same principle can be applied to light traveling in the ether wind. The time it takes for light to travel a distance of d against the wind, reflect from a mirror, and return is illustrated by the boat traveling in a river. The time it takes light to travel a distance of d perpendicular to the wind, reflect from a mirror and return, is represented by the boat traveling on a lake. There should be a distinct difference in travel time between light traveling parallel to the ether wind and perpendicular to it. Parallel to the wind means first traveling upstream against the wind and then downstream with the wind. Michelson hoped to measure this difference in time, or at least detect that there was a difference.

Based on this line of reasoning, Michelson designed and built a device that he called an interferometer. A picture of his interferometer is shown in Figure 27, taken from an 1881 paper that he published in the American Journal of Science.

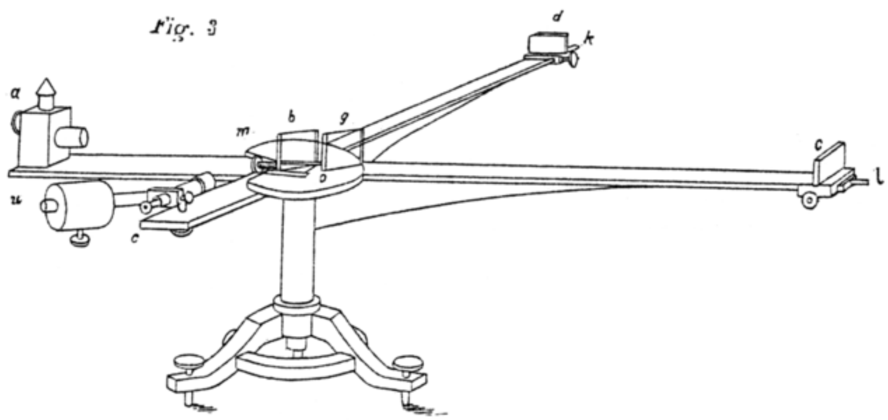

Figure 27 Michelson's first interferometer (source: Michelson)

An interferometer uses a half silvered mirror to split a beam of light into two perpendicular beams, illustrated as a green beam and a red beam in Figure 28. The green beam is diverted by the beam splitter and travels a distance of "d" to Mirror 1. At Mirror 1 it is reflected back through the beam splitter to a detector. In contrast, the red beam initially passes through the beam splitter and travels to Mirror 2, also a distance of "d" from the beam splitter. At Mirror 2 the red beam is reflected back to the beam splitter where it is diverted to the detector.

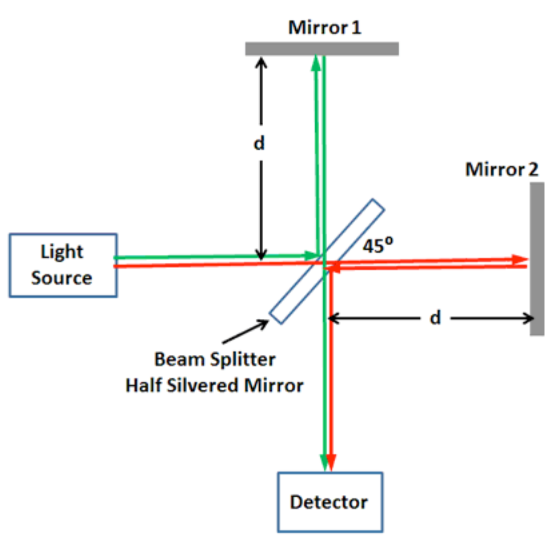

Figure 28 Interferometer (source: www.mpoweruk.com)

An interference pattern consisting of light and dark fringes (Figure 29) will develop if the time required for the red beam to reach the detector is different from that of the green beam. For example, if the red beam is traveling parallel the ether wind, it will take longer to reach the detector than the perpendicular green beam, as illustrated above. The width and number of fringes permits very delicate measurements to be made comparing the velocity of light rays traveling perpendicular to each other.

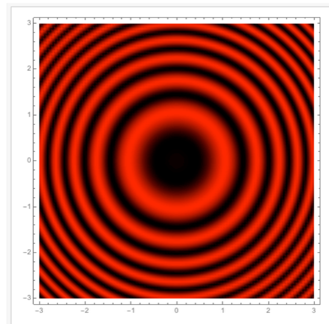

Figure 29 Michelson Interferometer interference pattern
(source: Wolfram Demonstrations Project)

Michelson's first attempt to detect the ether was conducted in Berlin using his new interferometer. However, his experiment failed. He did not detect any interference fringe patterns.

In 1883 Michelson resigned from the Navy to become Professor of Physics at the Case School of Applied Science in Cleveland, Ohio. Shortly after joining Case, using improved equipment, Michelson measured the speed of light to be 299,853 km/sec. This remained the accepted speed of light for nearly 45 years until Michelson refined his measurements at Mt. Wilson Observatory in the mountains above Pasadena, California.

While at Case, Michelson made another attempt to detect the presence of ether wind. This time he worked with Edward Morley (Figure 30). Morley was a professor of chemistry at Western Reserve University (WRU) which shared its campus with Case.

Figure 30 Edward W. Morley (source: Wikipedia)

Michelson and Morley designed an improved interferometer that they believed would be more than accurate enough to detect the presence of ether wind. The interferometer they built (Figure 31) was massive. Multiple mirrors were used to reflect light back and forth along the arms of the interferometer before entering the detector, increasing the path length to 11 meters (36 feet). The detector consisted of a small screen and microscope with which to view the anticipated fringe patterns. The interferometer was assembled on a stone slab over five feet square and 14 inches thick designed to isolate the interferometer from external vibrations. The massive assembly was then floated on a pool of mercury that served as a frictionless shock absorber to further isolate the interferometer from vibrations. The near zero friction of the mercury allowed the entire mechanism to rotate at a speed of up to 10 revolutions per hour. Once given a slight push it took hours for the rotating stone slab to come to a halt. To minimize thermal effects from changes in ambient temperature, the experiment was performed in the basement of Adelbert Dormitory, a brick building of considerable size at WRU. The dormitory was since renamed Pierce Hall and finally demolished in 1962.

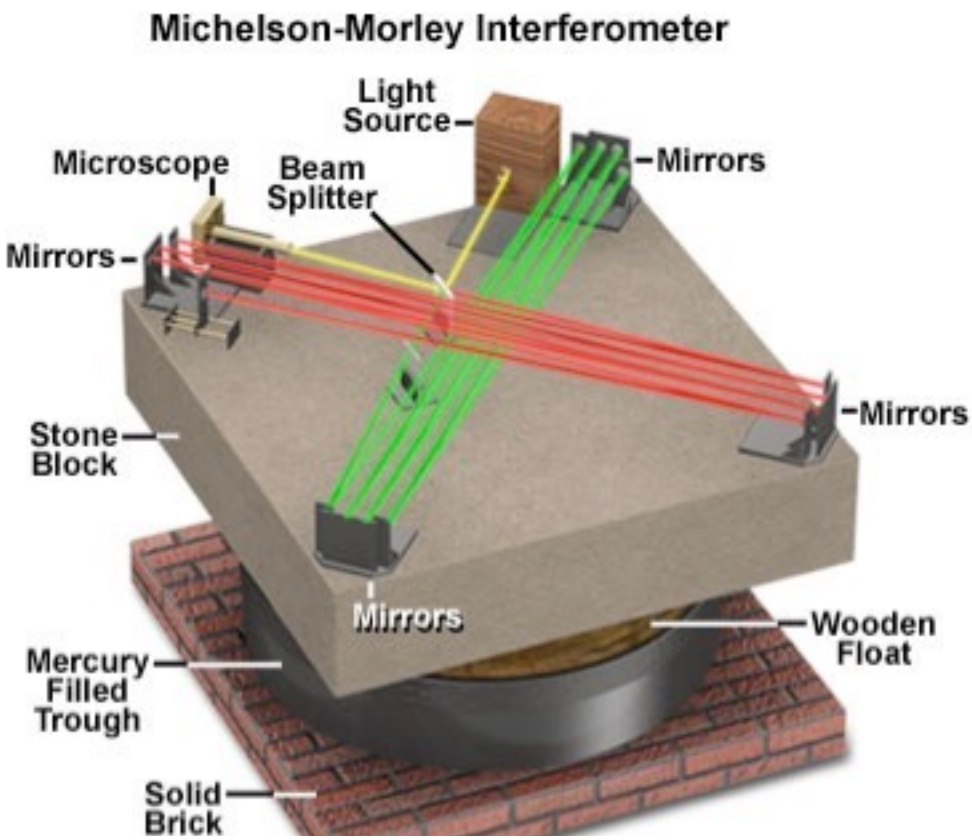

Figure 31 Michelson – Morley Interferometer (source: Wikipedia)

The experiment was conducted from April through July 1887. The expectation was that rotating the interferometer would cause it to pass through all possible angles with respect to the ether wind, inevitably causing one arm to turn into the wind while the other arm was perpendicular to it, producing interference fringes that could easily be viewed through the microscope. However, no interference fringes were detected.

The experiment became known as the most famous failed experiment in history because of its extreme importance in what occurred during the following years. Einstein wrote, "If the Michelson–Morley experiment had not brought us into serious embarrassment, no one would have regarded the relativity theory as a (halfway) redemption."

3.10 Einstein's Special Theory of Relativity and the Speed of Light

Einstein published his Special Theory of Relativity in 1905. The theory simply states that the laws of physics are the same for all observers in uniform motion. The laws of physics that we observe here on Earth are the same as on Mars, on some planet on the other side of our galaxy, they are the same everywhere in the universe. The theory is called special because it is limited to uniform motion. Einstein's more complex General Theory of Relativity, published in 1916, removes this restriction expanding the theory to include objects at rest as well as in accelerated motion. In addition, it refines Newton's law of universal gravitation providing a unified description of gravity as a geometric property of space and time.

According to Einstein's Special Theory of Relativity, Maxwell's equations apply everywhere. The speed c of electromagnetic waves, including light, as defined by Maxwell is equal to

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 299{,}792{,}458 \ \mathit{meters/second}$$

where

ε_0 = permittivity of free space = $8.85418782 \cdot 10^{-12} \ \frac{(coulomb)^2}{Newton\ meter^2}$

μ_0 = permeability of free space = $4\pi 10^{-7} \frac{Newton}{(coulomb/second)^2}$

Consequently, the speed of light in free space is a constant which is 299,792,458 meters per second.

In terms of Michelson and Morley's experiment, the speed of light is the same in all directions. It is the same parallel to the ether wind as it is perpendicular to the wind. The ether wind does not, can not, change the speed of light. Michelson and Morley did not see any fringe patterns when they conducted their experiment because the speed of light was the same on both legs of their very complex interferometer.

How does this correlate with our boat on a river? At the very slow speeds that we encounter in our daily lives, speeds do add and subtract from one another. But, at relativistic speeds, as we approach the speed of light, all speeds converge on the speed of light which is as fast as we can go.

The motion of all the waves that we are accustomed to (ocean waves, sound waves, etc.) are defined by the physics of the medium through which the waves travel. However, the electromagnetic waves predicted by Maxwell's equation are not defined by the physics of some underlying medium, an ether. The equations stand on their own. A time varying electric field induces a time varying magnetic field which in terns induces a time varying electric field and so on back and forth as the wave travels forever though the emptiness of free space. Einstein stated it clearly, there is no ether.

3.11 Michelson Mt Wilson Experiment

Michelson served three years as professor of physics at Clark University in Worcester, Massachusetts. In 1892 he was appointed head of the physics department at the newly formed University of Chicago, a position that he held until his retirement in 1929. In 1907 Michelson was awarded the Nobel Prize for his work in spectroscopic and metrological investigations performed with the precision optical instruments that he developed. He was the first American to be awarded a Nobel Prize in physics.

When World War I broke out, Michelson rejoined the Navy developing spectroscope and rangefinder equipment. Michelson returned to the University of Chicago following the war. In 1920 Michelson used a 6-meter interferometer attached to the 100-inch Mt Wilson telescope to measure the diameter of the star Betelgeuse. The diameter of Betelgeuse was found to be 386,160,000 km, about the size of the orbit of Mars around the Sun. This was the first accurate measurement of the size of a star.

Michelson's longtime assistant at Mt. Wilson was Francis G. Pease (Figure 32). In 1908 Pease became an astronomer and instrument maker at the Mt. Wilson Observatory. Among his designs was the 100-inch telescope at the observatory. Pease would later become involved in designing the 200-inch Hale Telescope at the Mount Palomar Observatory near San Diego, California.

Figure 32 Francis G. Pease, 1881-1938 (source: Wikipedia)

From 1923 through 1927 Michelson and Pease, in effect, fulfilled Newcomb's vision of measuring the speed of light over a long distance. Newcomb had suggested conducting the experiment in the Rocky Mountains or the California Sierra Nevada Range. Instead, Michelson performed his measurements in the San Gabriel Mountains of Southern California. For his measurement Michelson used an eight-sided steel mirror rotated at 528 revolutions per second. The rotating mirror was installed at the Mt. Wilson Observatory. A fixed mirror was positioned 35 km away on Mount San Antonio. The U.S. Coast and Geodetic Survey Agency spent two years of painstaking work measuring the distance between the rotating and fixed mirrors. The accuracy of the final measurement had an error of less than 2.5 cm. Using this set up, Michelson determined the speed of light to be 299,796 ± 4 km/s which was, at the time, the most accurate measurement of the speed of light ever performed. In comparison, today's value for the speed of light is 299,792.458 km/sec.

After Michelson's death in 1931, Pease continued their work at Mt. Wilson. Pease made numerous measurements over several years using a modified interferometer. The final value that he obtained for the speed of light was 299,774 km/sec.

3.12 Microwave Speed of Light Measurements

During the 1940s and 1950s British physicists Louis Essen and Keith Davy Froome, working separately, employed microwaves techniques to more precisely measure the speed of light (electromagnetic radiation).

3.12.1 Louis Essen

Essen (1908 – 1997) joined the U.K. National Physical Laboratory (NPL) in 1929. Initially he worked on time and frequency standards. Beginning in 1932 he concentrated his efforts on quartz oscillators, eventually developing the quartz ring clock. The high stability of this clock permitted it to be used in astronomical observatories including the Royal Observatory, Greenwich, and the US Naval Observatory.

During World War II, Essen solved instability problems in high frequency communication systems and developed special microwave cables required for short wave radar. This was cutting edge technology requiring Essen to develop new types of measuring equipment including the cavity resonance wavemeter. This device became widely used in industry and international calibration.

Following the war, Essen realized that the cavity resonator could be used to measure the speed of light with greater accuracy than had been previously done. In 1946 and 1947 he conducted experiments at the National Physical Laboratory to do just that.

Essen's experiment consisted of injecting a signal from a variable frequency microwave oscillator into a hollow closed copper cylinder and noting the frequency at which resonance occurred within the cavity. Knowing this, and the exact dimensions of the cavity, the speed of light could be calculated. The results of his experiment were documented in the paper titled "The velocity of propagation of electromagnetic waves derived from the resonant frequencies of a cylindrical cavity resonator" published by The National Physical Laboratory on 4 December 1947.

A diagram of the equipment that he used in his experiments is shown in Figure 33.

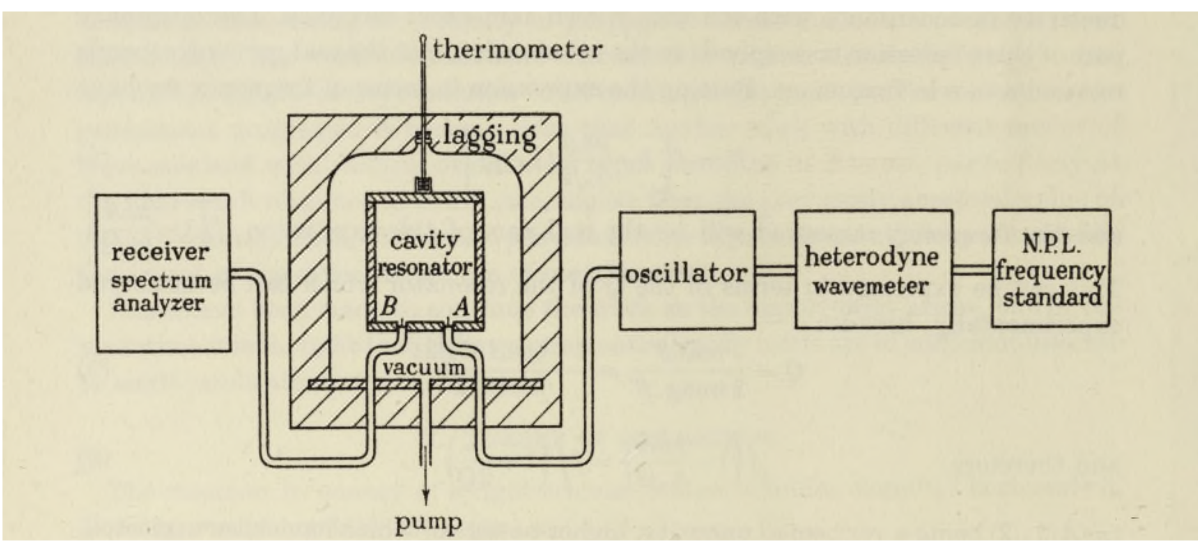

Figure 33 Essen Cavity Resonator Experiment (source: Essen)

Using this apparatus the speed of light, v_0 , can be calculated from the equation

$$v_0 = \frac{f_{lmn}(1 + 1/2Q)}{\sqrt{[(r/\pi D)^2 + (n/2L)^2]}}$$

where

f_{lmn} = the frequency of the microwave signal for a particular mode of resonance

D = the diameter of the cylindrical cavity resonator

L = the length of the cavity resonator

r = a constant for a particular mode of resonance

n = the number of half wavelengths occurring in the resonator

Q = a quality factor.

For this experiment the cavity was formed by milling out a cylinder 7.4 cm in diameter (D) and 8.5 cm in length (L) from a block of solid copper.

The best results were obtained using resonant modes identified as E_{010} and E_{011}.

The quality factor Q of the resonator was measured by observing the change in frequency, from the resonant value f, required to reduce the magnitude of the detected microwave signal to $1/\sqrt{2}$ of its peak value. If this change is δf then

$$Q = \frac{f}{2\delta f}$$

The values of Q obtained for the constructed cavity were

Q = 18,000 @ E_{010} and

Q = 14,000 @ E_{011} .

The cavity resonator was coupled to a microwave oscillator and a receiver by means of probes A and B as illustrated in Figure 33. As the frequency of the oscillator was varied, a sharp increase in the amplitude of the signal detected by the receiver occurred when the oscillator frequency corresponded to one of the cavity's resonant frequencies. The oscillator frequency was measured by a heterodyne wave-meter and recorded when this occurred.

The resonator was enclosed under a bell-jar and evacuated for 8 hours prior to performing an experiment. In addition, the vacuum-tube electronic equipment was turned on for several hours before an experiment to allow the equipment to reach thermal stability.

The results of Essen's experiment are tabulated in the following table.

Mode of Resonance	Correction Factor (1+1/2Q)	Constant r	Measured Frequency f MHz	v_0 (km/s)
E_{010}	1.000028	2.404825	3,101.25	299,793
E_{011}	1.000035	2.404825	3,563.80	299,791
E_{010}	1.000028	2.404825	3,101.28	299,796
E_{011}	1.000035	2.404825	3,563.77	299,789

The mean value of velocity that Essen arrived at was 299,792 km/s. However, this value was substantially higher than previous optical measurements. In 1935 Pease and Pearson had measured $299{,}774 \pm 11\ km/s$. In 1941 Anderson measured $299{,}776 \pm 14\ km/s$. Consequently, Essen's results were fiercely criticized. Essen refined his apparatus and repeat his measurement in 1950, this time obtaining a value of $299{,}792.5 \pm 1\ km/s$. This value was accepted and in 1957 adopted by the 12th General Assembly of the Radio-Scientific Union as the official speed of light.

It is interesting to note that Essen was one of developers of the first atomic clock (see "Measuring Time" under Related Topics).

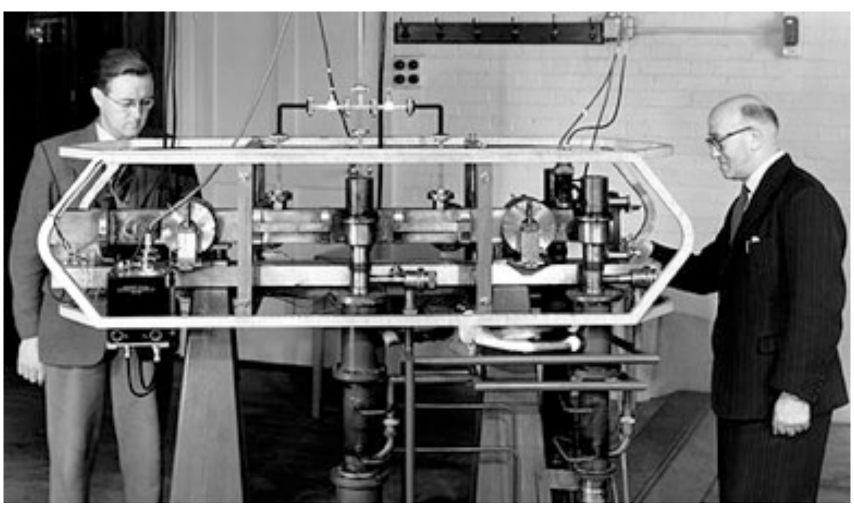

Figure 34 Louis Essen (right) and Jack Parry (left) with the original National Physics Lab (NPL) Cesium clock. (source: Lombardi)

3.12.2 Keith Davy Froome (1921 – 1995)

Keith Davy Froome was born in Constantinople, Turkey in 1921 where his father was a Captain in the Royal Army Ordnance Corps in charge of an ammunition depot. Froome's family moved back to Britain when he was 18 months old. He spent his early childhood at Wellington in Shropshire. Froome studied at University College London from 1939 to 1941 graduating with a degree in Special Physics. The college was evacuated to Bangor N. Wales during the early part of World War II. From 1945 to 1947 Froome studied transient arc discharges at Imperial College London where he was awarded his PhD.

Froome joined the National Physical Laboratory (NPL) in 1949 as a Senior Scientific Officer. In 1951 he began his collaboration with Dr Essen on the study of refractive indies for air at radio and microwave frequencies. Froome was promoted to Deputy Chief Scientific Officer at the NPL in 1969,

In 1958 Froome used a microwave interferometer, Figure 35, to measure the speed of electromagnetic waves at a frequency of 72.006 GHz, a wavelength of approximately 4 mm.

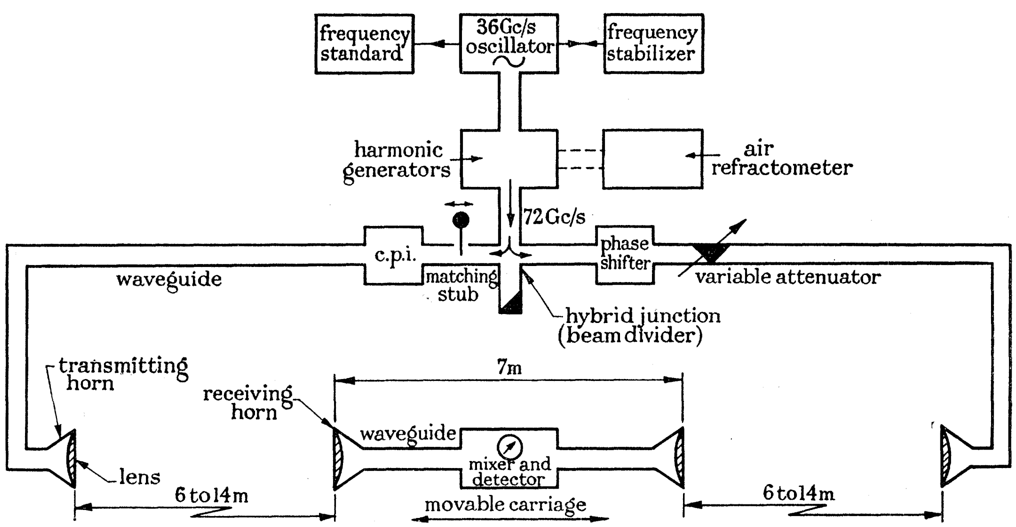

Figure 35 Froome 72 GHz Interferometer (source: Froome)

The microwave signal was generated by a 36 GHz oscillator and boosted to 72 GHz using a harmonic generator. Following the harmonic generator, the 72 GHz signal was split into two beams each traveling through identical wave guides to transmitting horns spaced 27 meters apart. A seven meter movable carriage, supporting two receiving horns, a mixer and a detector, was placed between the two transmitting horns as illustrated in

Figure 35. Moving the carriage changed the path lengths of the two beams creating interference patterns when the two received signals were mixed together. Minimums occurred in the interference patterns with every half-wavelength displacement of the carriage. The carriage was moved through 970 half-waves (roughly 2 m) and the distance between minimums measured. The wavelength value so obtained, 4.163 mm, was multiplied by the signal's 72.006 GHz frequency to give its phase velocity, that is

$$v = \lambda \cdot f$$

where

v = the velocity of the measured microwave signal,

f = the frequency of the microwave signal, and

λ = the measured wavelength of the signal derived from the interference patterns.

Froome's results are summarized in the following table.

Separation between transmitting & receiving horns (cm)	**Phase velocity v_0 (km/s)**
629.5	299,792.513
751.5	299,792.529
875.0	299,792.476
999.0	299,792.414
1,120.5	299,792.478
1,247.5	299,792.588
1,367.5	299,792.512

with a mean value of $v_0 = 299{,}792.501 \pm 0.059\ km/s$

Froome published his results March 25, 1958.

3.13 Laser Measurements of the Speed of Light

Before the advent of lasers, infrared and visible light were measured in terms of their wavelengths (λ) with a high degree of accuracy. However, the frequency (f) of the light being measured could only be determined by multiplying its wavelength by the then accepted value for the speed of light (c) . That is,

$$f = \frac{c}{\lambda}$$

Development of lasers offered the possibility of directly measuring the speed of light. A laser provides a coherent frequency source that, in theory, permits both its wavelength and frequency to be measured. Unfortunately, the technology for directly measuring frequencies in the infrared and visible spectrum was not available. In the late 1970s, Kenneth Evenson (Figure 36) and his colleagues at the National Institute of Standards and Technology (NIST) in Bolder, Colorado solved this problem.

Direct frequency measurement of a wave is done by counting the number of wave cycles that occur in a given period of time. Available electronic circuitry permitted direct cycle counting up to about 500 MHz, far short of what was needed. To directly measure the frequency of a laser beam it was necessary to mix the beam with a known extremely high frequency signal. The mixing process would produce two new signals, one equal to the sum of the beam and known signal frequencies and the other equal to their difference. If the frequency of the known signal was close to that of the laser beam, then the difference signal would be low enough in frequency to be counted. The problem was that generating a known signal at such a high frequency had never been done. To solve that problem a chain of harmonic generators was constructed driven by the super accurate microwave output of a cesium clock. The frequency of the signal at the output of the generator chain was too high to be directly measured. However, it was an exact known multiple of the cesium clock. Consequently, the frequency of that signal was known. Mixing it with the laser beam was accomplished using a special metal-metal point contact diode. The catwhisker forming the diode acted as a “long wire antenna” at the laser beam frequency. When the diode was irradiated by the laser, the beam mixed with the known frequency signal producing the desired low frequency difference signal whose frequency could be counted. The frequency of the laser beam was then equal to frequency of the known signal plus the frequency of the difference signal.

Using a methane stabilized He-Ne laser, Evenson and his team successfully measured both the wavelength and frequency of the laser, the product of the two being the speed of light. The value that they obtained was 299,792,458.0 m/s $\pm 4 \; x \; 10^{-9}$, one hundred times more accurate than the previously accepted value. That is, their value was measured to two additional decimal points.

In 1983 the Seventeenth General Congress on Weights and Measures officially defined the speed of light as being 299,792.458 kilometers per second. In addition, it redefined the meter to be the distance that light travels during a time interval of 1/299,792,458 seconds.

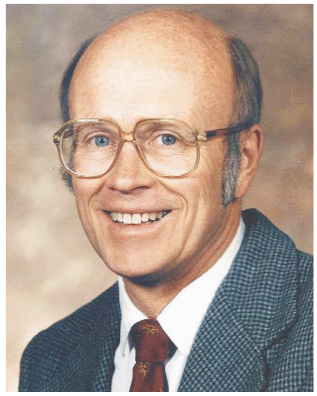

Figure 26 Kenneth M Evenson (source: Physics Today – Scitation)

3.14 Summary

The speed of light has been measured at least 163 times utilizing a wide variety of different techniques by more than 100 investigators since Roemer's initial 1676 measurement. The following table summarizes the results obtained by some of these investigators.

Date	Investigator	Method	Estimated km/s
1667	Galileo Galilei	Covered Lanterns	333.5
1676	Ole Roemer	Jupiter's Moons	211,932
1726	James Bradley	Stellar Aberration	301,155
1834	Charles Wheatstone	Rotating Mirror	402,336
1838	François Arago	Rotating Mirror	
1849	Armand Fizeau	Rotating Wheel	313,274
1862	Leon Foucault	Rotating Mirror	298,000
1868	James Clerk Maxwell	Theoretical Calculations	284,000
1875	Marie-Alfred Cornu	Rotating Mirror	299,990
1879	Albert Michelson	Rotating Mirror	299,910
1888	Heinrich Rudolf Hertz	Electromagnetic Radiation	300,000
1889	Edward Bennett Rosa	Electrical Measurements	300,000
1890s	Henry Rowland	Spectroscopy	301,800
1907	Edward Bennett Rosa and Noah Dorsey	Electrical Measurements	299,788
1923	Andre Mercier	Electrical Measurements	299,795
1926	Albert Michelson	Rotating Mirror (Interferometer)	299,796
1928	August Karolus and Otto Mittelstaedt	Kerr Cell Shutter	299,778
1932 to 1935	Pease and Pearson	Rotating Mirror (Interferometer)	299,774
1947	Louis Essen	Cavity Resonator	299,792
1949	Carl I. Aslakson	Shoran Radar	299,792.4
1951	Keith Davy Froome	Radio Interferometer	299,792.7
1973	Kenneth M. Evenson	Laser	299,792.458

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