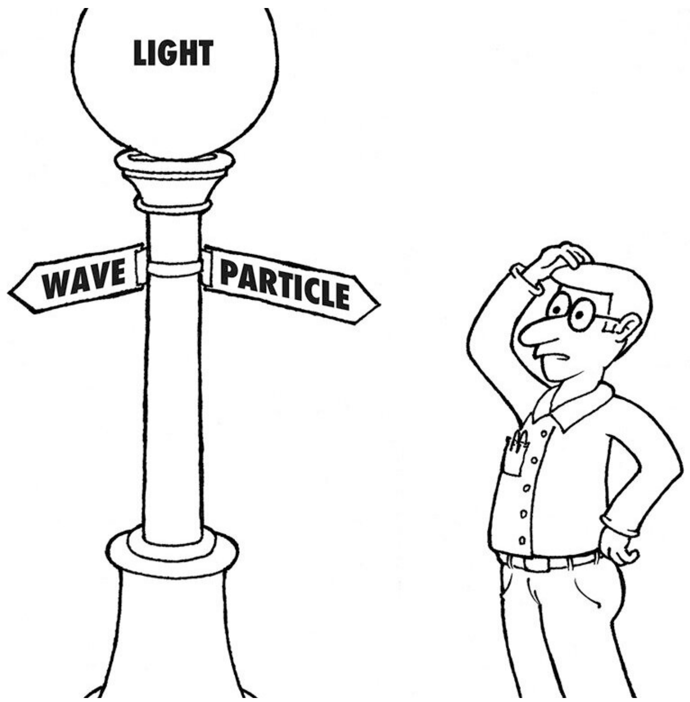
Wave Particle Duality



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5 Introduction

By the mid 1800s it was proved conclusively that light is a wave, finally putting to an end the wave vs particle debate that raged for well over a hundred years. A scant 50 years later the whole issue reopened as the first concepts of quantum mechanics began to emerge. However, this time it was different. The answer to the question of whether light is a wave or a particle is that light is **both** a wave and a particle. Light behaves as a wave as it propagates from one place to another, reflects, refracts, and interferes both constructively and destructively with other waves. Light behaves as a particle, a photon of light, when interacting with matter. A photon being a particle with zero mass that travels at the speed of light.

At the end of the nineteenth century the field of physics consisted of classical mechanics (initially defined by Newton), Maxwell's theory of electromagnetics, and thermodynamics first formulated by Carnot. The overwhelming success of these disciplines in describing all known physical phenomena lead people to believe that the ultimate description of nature had been achieved.

At the beginning of the 20th century the concepts of classical physics were seriously challenged on two fronts:

- Relativistic theories, and
- The discoveries of the microscopic world

Einstein's 1905 theory of special relativity showed that the laws of classical mechanics were not valid at very high speeds approaching the speed of light.

At the beginning of the 20th century new technologies were developed that, for the first time, allowed the structure of atoms to be investigated. It quickly became evident that classical physics failed miserably in explaining phenomena at the atom and subatomic levels. The new field of quantum mechanics, capable of explaining these phenomena, slowly emerged.

In 1900 Max Planck introduced the concept of a quantum of energy to explain blackbody radiation. Previously it was believed that the spectrum of energy radiated by an object was continuous. That is, an object could possess any amount of energy. Planck suggested instead that the energy of an object could only exist at discreate harmonically related energy levels $E_n = n \cdot h \cdot f$. The energy radiated by an object was not continuous. Using this concept Planck was able to develop a mathematical expression that precisely produced blackbody energy distribution curves that previously could only be empirically constructed.

In 1905 Einstein used Planck's concept of a quantum of energy to explain the photoelectric effect discovered by Hertz in 1887. Einstein proposed that light is composed of tiny particles of energy which he called photons. A photon has zero mass but travels at the speed of light. The energy of a photon is

$$E = h \cdot f$$

where h is Planck's constant and f is the frequency of light energy carried by the photon. The energy of a photon thus increases as its frequency increases. This property allowed Einstein to explain why light toward the high frequency blue end of the spectrum was able to knock electrons out of a metal plate while red light was not able to do so, one of the perplexing problems of the photoelectric effect.

In 1913 Niels Bohr used Rutherford's 1911 discovery of the atomic nucleus, Planck's quantum of energy theory, and Einstein's concept of photons to derive his model of the hydrogen atom. In his model Bohr argued that atoms can only exist at discrete energy levels or states. Furthermore, atoms can only radiate and absorb light in discrete amounts of energy $h \cdot f$ associated with the transition of an atom from one energy state to another.

Arthur Compton provided a conclusive confirmation of the particle nature of light in an experiment that he performed in 1923. In his experiment he scattered a beam of X-rays off of stationary electrons confirming that X-ray photons behave like particles with momenta of

$$p = \frac{hf}{c}$$

De Broglie proposed in 1923 that not only do light waves exhibit particle behavior, but conversely material particles display wave-like behavior. This concept was confirmed experimentally by Davisson in 1927 thus establishing the wave-particle duality.

5.1 Blackbody Radiation

All objects at temperatures above absolute zero ($> -273.15°\mathrm{C}$) emit heat in the form of electromagnetic radiation. The hotter the object the shorter the wavelength of its radiated energy (the higher its frequency since $f = c/\lambda$). If an object is hot enough, the wavelength of its emitted electromagnetic energy will be in the visual part of the spectrum. For example, an iron bar at room temperature is not visible in a dark room. But, if the bar is heated to several hundred degrees it will glow red in the dark room. If heated to a very high temperature it will become white hot as illustrated in Figure 1.

Intuitively this makes sense. The temperature of an object is a measure of how rapidly its atoms are vibrating back and forth. The higher the temperature, the greater the vibrations. Atoms are composed of charged particles (electrons and protons). These charged particles are confined to their respective atoms and thus also vibrate back and forth. Moving charged particles which are continuously accelerating and decelerating produce electromagnetic waves. Thus, it is no surprise that all objects with temperatures above absolute zero radiate electromagnetic energy. The higher the object's temperature, the higher the frequency of its electromagnetic waves. However, at a temperature of absolute zero, all atomic vibrations stop. Consequently, at absolute zero atoms no longer radiate electromagnetic energy. This is an extreme theoretical case. The laws of quantum mechanics state that all objects, particles, atoms, etc. have a minimum energy level greater than absolute zero. This minimum permitted energy is called the object's ground state energy. According to quantum mechanics, an object can approach absolute zero but never actually reach it since its ground state energy must always be greater than zero. Consequently, everything must radiate some sort of heat in the form of electromagnetic radiation.

Figure 1 Iron bar first turns red then white hot when heated (source: sutterstock)

Extensive experimentations in the late 1800s showed that the wavelength and intensity of an object's radiation depended on its temperature. The hotter it gets the shorter the wavelength and the greater the intensity of its radiation. A heated iron bar first turns a dull red, then yellow, and finally a brilliant white as illustrated in Figure 1. However, all attempts to derive equations relating temperature to the wavelength and intensity of an object's radiation failed. The equations inviably blew-up predicting that the energy of an object would become infinite as the wavelengths of its emitted light approached the ultraviolet part of the spectrum. This was of course a ridicules conclusion. For this reason, the problem became known as the ultraviolet catastrophe.

The problem was finally solved in 1900 by German physicist Max Planck (Figure 2). Planck derived the following equation which defines the intensity and wavelength of an object's electromagnetic radiation based solely on the object's temperature.

$$I_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

Figure 2 Max Plank (1858 -1947) (source: sciencephoto.com)

A graph of this equation for three different values of temperatures T is shown in Figure 3 verses wavelengths λ. The horizontal axis of the graph is wavelength λ measured in micro-meters μm. The vertical axis is the intensity of the radiated energy measured in energy per unit time (power) per unit area per unit wavelength. In Figure 3 however the vertical axis is simply an arbitrary scale of 0 – 14 since it is the shape of the curve that is of primary interest in Figure 3.

The blue, green, and red traces in Figure 3 are obtained from Planck's equation at temperatures of T = $5000, 4000, and\ 3000\ °K$. Notice that all three traces go to zero at long wavelengths ($> 3\mu m$) and also at short wavelengths ($< 0.25\ \mu m$) avoiding the ultraviolet catastrophe. The black trace on the right, labeled Classical Theory, erroneously shows the energy radiated by an object going to infinity as its wavelength gets shorter.

The traces in Figure 3 are distribution curves. The blue trace represents the distribution curve for an object with a temperature of $5{,}000\ °K$. The curve peaks at a wavelength of approximately 0.58 μm, meaning that most of the atoms in this $5{,}000$ ° object vibrate at this wavelength. However, a significant number of its atoms also vibrate a wavelength of 1 μm while hardly any vibrate at wavelengths longer than 3 μm. On the left side of the energy peak, a considerable number of atoms vibrate at roughly 0.4 μm while essentially none vibrate at wavelengths shorter than 0.2 μm.

The intensity of the radiated energy drops as the temperature of an object decreases. For example, the peak intensity of a $4{,}000\ °K$ object is considerably less than that of an object at $5{,}000$ °. However, the shapes of the two curves are similar, as they must be since they are both derived from the same equation. The only difference being the object's temperature. The peak intensity of a $3{,}000\ °K$ object is even less.

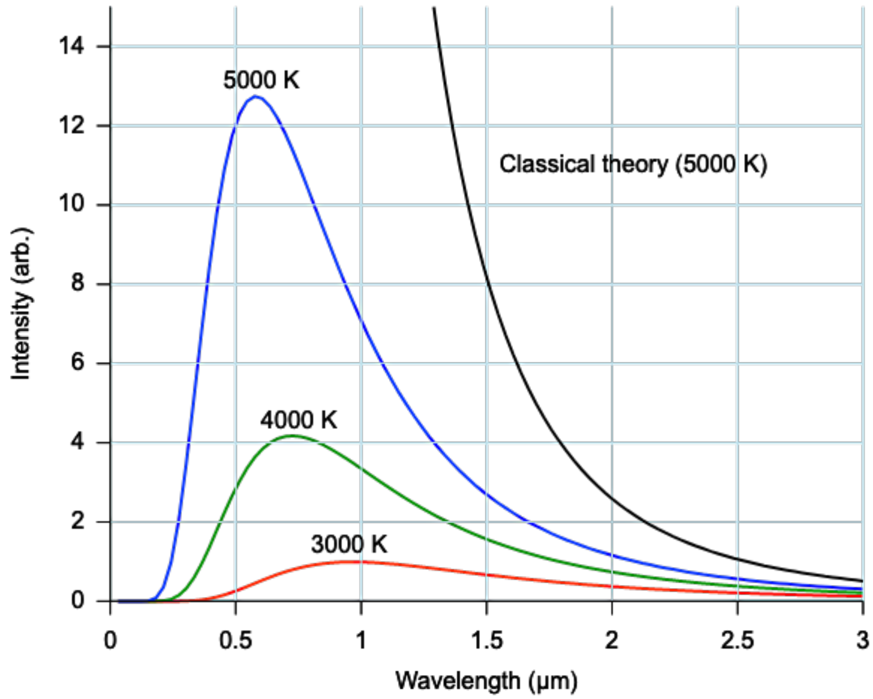


Figure 3 Plot of Planck's equation for various temperatures and wavelengths (source: en-academic.com)

Objects that adhere to Planck's equation are defined as blackbody objects. Specifically, a blackbody

1) Absorbs all electromagnetic energy incident on it, that is, it does not reflect any light.

2) The spectrum of electromagnetic energy radiated by a blackbody depends **only** on the temperature of the blackbody, not what it is made of.

3) The intensity of the electromagnetic energy radiated by a blackbody is completely defined by Planck's equation, peaking in the middle and going to zero at both long and short wavelengths.

A blackbody does not have to be black in color. For example, the Sun and the stars approximate blackbodies even though their colors range from red, to yellow, to blue.

A person does not satisfy the conditions required to be a blackbody because people reflect light. The light that they reflect is how we are able to see them, the color of their cloths, etc. However, in a completely dark room there is no light to reflect. In that environment a person does radiate as a blackbody, with the peak of its blackbody radiation occurring in the infrared region of the spectrum as shown in Figure 4 and Figure 5.

Figure 4 Blackbody radiation from a person (source: NASA)

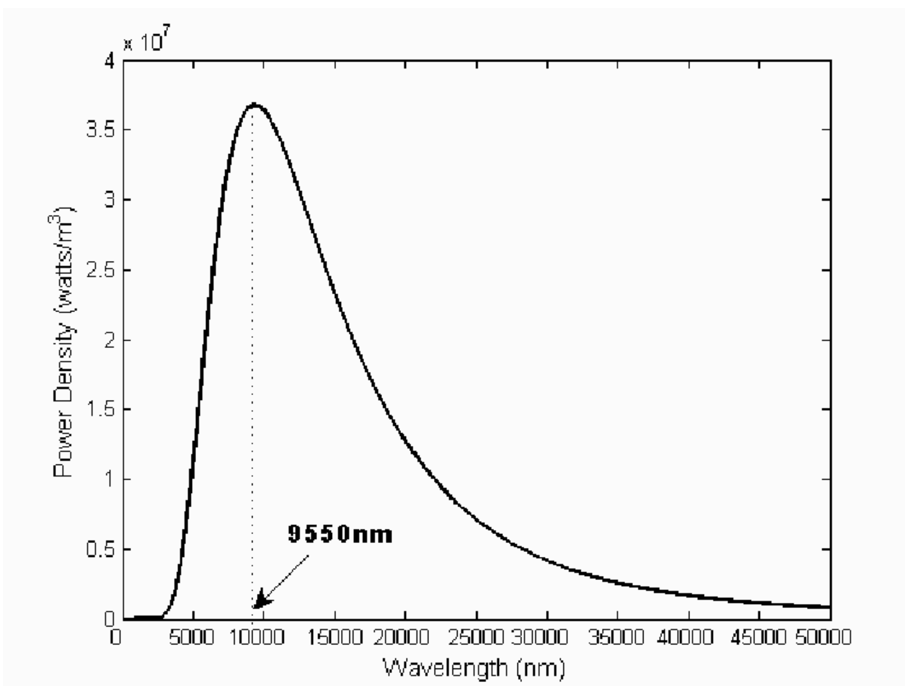


Figure 5 Blackbody radiation curve for a human (source: researchgaate.net)

Notice in Figure 5 that the blackbody radiation of a human peaks at a wavelength of 9,550 nm (9.55 μm) far to the right of the distribution curves shown in Figure 3.

A person, of course, always radiates as a blackbody. But in sunlight the blackbody radiation of a person is overwhelmed by reflected light.

5.1.1 Planck's Assumptions

To deal with the ultraviolet catastrophe Planck made two assumptions:

1. An object that radiates electromagnetic waves at some fundamental frequency f also radiates energy at all harmonics of f. If n is a particular harmonic then the permitted harmonics are all integers such that n = 1, 2, 3, etc. However, an object can only radiate energy at its fundamental and integer harmonic frequencies. It can not, for example, radiate energy at a frequency of 2.5 f. Furthermore, Planck defined the energy E_n associated with a particular harmonic n to be

$$E_n = n \cdot h \cdot f$$

   where h is Planck's constant with a value of = 6.626×10^{-34} joule / Hz.

2. The probability of a harmonic n occurring decreases exponentially as n increases in value.

Prior to Planck's equation it was assumed that the energy spectrum of the vibrating atoms composing an object was continuous. That is, and individual atom could possess any amount of energy. Planck instead proposed that the atoms of an object could only vibrate at discreate harmonically related energy levels E_n. This was not a completely novel idea. Spectral analysis of elements in the early 1860s by Gustav Kirchhoff and Robert Bunsen revealed that every element emits and absorbs light only at very specific wavelengths.

In Planck's equation

$$E_n = n \cdot h \cdot f = \frac{n\,h\,c}{\lambda_n}$$

the lowest energy level that the atoms of an object can have occurs at n = 1 which is an atom's ground state. Furthermore, every atom must have a non-zero ground state energy, so n can not equal zero.

The term

$$E_1 = \frac{h\,c}{\lambda_1}$$

is the ground state quantum of energy for an atom, with λ_1 being its ground state wavelength. According to Planck, all energy levels which an atom is capable of obtaining must be an integer multiple n of its ground state energy such that

$$E_n = n \cdot \frac{h\,c}{\lambda_1}$$

Planck's constant h is just a number that is given to us by nature. It is not predicted by any theory of science. It must be experimentally measured like the mass of an electron or the gravitational constant. Planck's constant h is an incredibly small number.

$$h = 6.626 \, x \, 10^{-34} \, joule/Hz$$

Planck's proposed quantization of energy poses an immediate problem. If the energy levels that an atom can attain are restricted to

$$E_n = n \cdot \frac{h\,c}{\lambda_1}$$

how can the blackbody distribution curves be continuous? Shouldn't the distribution curves only have values at $n \cdot \lambda_1$? The answer is that the distribution curves are quantized. However the distance d between a permitted wavelength λ_n and the next allowed wavelength of λ_{n+1} is

$$d = \lambda_{n+1} - \lambda_n = \left[\frac{(n+1)hc}{E_{n+1}}\right] - \left[\frac{nhc}{E_n}\right]$$

if

$$E_{n+1} \approx E_n$$

then

$$d = \frac{[(n+1)hc] - [nhc]}{E_n} = \frac{hc}{E_n}$$

which is an extremely small distance (on the scale of atomic dimensions) because Planck's constant h is such an incredibly small number. Thus, while the blackbody distribution curves are quantized, the spacing between allowed wavelengths is so small we can not see the quantization. The distribution curves appear continuous.

The equation

$$E_n = \frac{n\,h\,c}{\lambda}$$

has another problem. It exhibits the ultraviolet catastrophe. Each harmonic n = 2, 3, 4, etc. has a progressively higher energy than the object's ground state E_1. Consequently, the energy of an object becomes infinite at very short ultraviolet wavelengths.

Planck solved this problem with his second assumption. The probability of a harmonic n occurring decreases exponentially as n becomes large. So, after a certain point, the probability of an atom attaining a very high harmonic frequency

$$nf_1 = \frac{nc}{\lambda_1}$$

decreases increasing fast, causing the probability to become so vanishing small that for all practical purpose we can consider it to be zero.

5.1.2 Examining Planck's Equation

Planck's great achievement was not only avoiding the ultraviolet catastrophe, but also developing an equation which produced the blackbody wavelength vs intensity curves that had previously only been experimentally determined. The ultraviolet catastrophe was avoided by defining a quantum of energy

$$E = \frac{h\,c}{\lambda},$$

Notice in Planck's equation

$$I_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

$$I_\lambda(T) = \left[\frac{hc}{\lambda}\right] \cdot \frac{2c}{\lambda^4(e^{hc/\lambda kT} - 1)}$$

that the quantum term

$$\frac{hc}{\lambda}$$

appears not only in the main part of the equation but also in the exponential term

$$e^{hc/\lambda kT}$$

as well.

The remaining terms appearing in the equation are:

I_λ = the intensity of the electromagnetic energy radiated by a blackbody, specifically

I_λ = energy per unit time (power) per unit area per unit wavelength

T = the blackbody temperature in degrees kelvin

λ = the wavelength of the radiated electromagnetic wave

c = the speed of light = 299,792.458 km per second

h = Planck's constant = 6.626×10^{-34} joule-seconds

k = Boltzmann's constant = 1.381×10^{-23} joules per kelvin

Notice that 3 of the 5 parameters in this equation are constants (c, h, and k). Only temperature T and wavelength λ are variables.

Each of the stars in Figure 6 is a blackbody absorbing all light incident upon it and radiating a spectrum of light determined entirely by its temperature. The very hot blue star has a temperature of $12{,}000\ °K$ which fixes the value of T in Planck's equation. With T fixed, the equation traces out the blackbody distribution curve for the blue star as wavelength λ, the only remaining variable in the equation, is varied from from 2,000 to 100 nm.

The term $2hc^2/\lambda^5$ in the numerator of Planck's equation goes to zero as λ becomes large causing the entire equation $I_\lambda(T)$ to go to zero at long wavelengths as shown in Figure 6.

Also, as λ becomes large the exponent $hc/\lambda kt$ in the denominator becomes increasingly small. In general, $e^x = 1 + x$ for small values of x. Thus as λ becomes large (long) Planck's equation reduces to

$$I_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} \cong \frac{2hc^2/\lambda^5}{1 + \frac{hc}{\lambda kT} - 1} = \frac{2hc^2/\lambda^5}{\frac{hc}{\lambda kT}} = 2ckT/\lambda^4$$

driving the equation further toward zero. This is an expected result since

$$I_\lambda(T) = \frac{2ckT}{\lambda^4}$$

is the Rayleigh–Jeans law developed in 1900. While this equation is correct for long wavelengths, it blows up (goes to infinity) as wavelengths become shorter, a problem pointed out by Rayleigh and Jeans. This equation, shown as the black curve labeled Classical Theory on the far right side of Figure 3, exhibits the ultraviolet catastrophe.

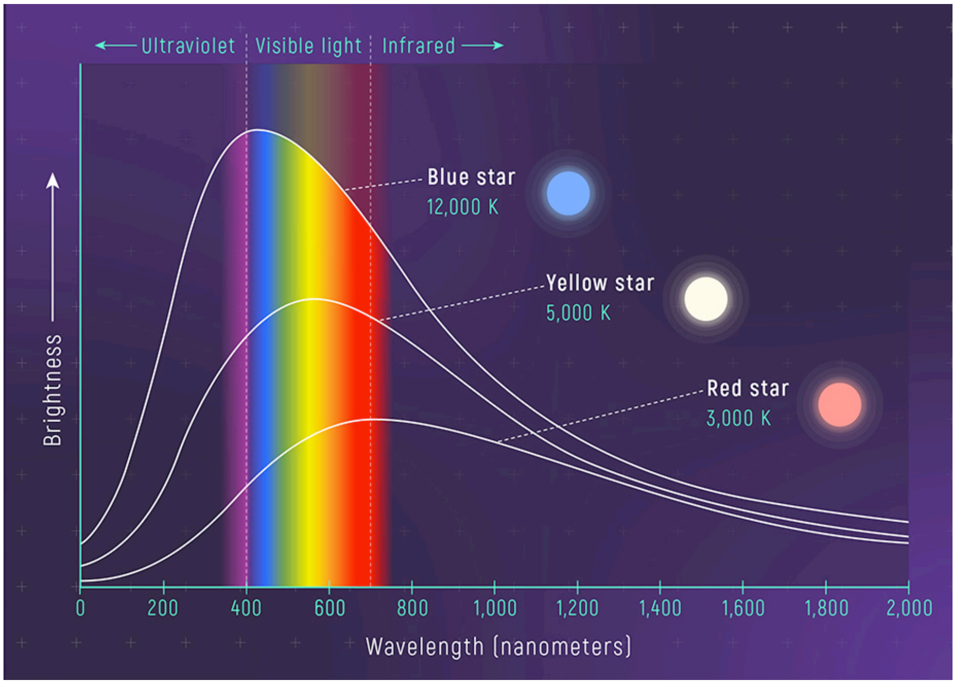


Figure 6 Blackbody spectrum of stars (source: NASA)

In Planck's equation

$$I_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

as λ becomes small (shorter in wavelength) the term $2hc^2/\lambda^5$ in the numerator becomes infinitely large. However, Planck's equation takes care of this problem. As λ becomes small the term $e^{hc/\lambda kT}$ in the denominator becomes very large very fast, overwhelming the term $2hc^2/\lambda^5$ and causing the intensity $I_\lambda(T) \to 0$, as illustrated in Figure 6, thus avoiding the ultraviolet catastrophe.

The wavelength at which the intensity of a blackbody curve peaks is determined by differentiating Planck's equation with respect to λ and setting the result equal to zero. When this is done the result is

$$\lambda_{peak} = \frac{b}{T}$$

which is in fact Wein's equation developed by Wilhelm Wien in 1893 based on thermodynamic arguments. The parameter b is a constant of proportionality called Wien's displacement constant. In retrospect (Wien's equation came first) Wien's displacement constant b is derived from the combination of constants $c, h, and\ k$ in Planck's equation after differentiation.

$$b = 2.897771955 \ldots x\ 10^{-3}\ m \cdot K \cong 2898\ \mu m \cdot K$$

when temperature is measured in degrees kelvin.

Consequently, as the temperature of an object gets hotter the peak in its blackbody curve moves toward the ultraviolet region of the spectrum. This is clearly shown in Figure 6. The intensity curve for the very hot blue star, at a temperature of over $12{,}000\ °K$, peaks in the blue segment of the spectrum, which is why the star appears to be blue. In contrast, a yellow-white star like our Sun, with a temperature of around $5{,}000\ °K$, peaks in the yellow part of the spectrum, specifically at

$$\lambda_{peak} = \frac{b}{T} = \frac{2898\ \mu m \cdot K}{5{,}000\ °K} = 580\ nm$$

which is to the right of the blue star shown in Figure 6. Finally, a cool red star with a temperature of around $3{,}000\ °K$ peaks even further to the right in the red part of the spectrum.

The fact that Planck's equation reduces to the Rayleigh–Jeans Law at long wavelengths and Wein's equation when wavelengths become short is not a coincidence. It was recognized at the time that the Rayleigh–Jeans Law was correct at long wavelengths but failed miserably as wavelengths became short. In contrast Wein's equation was correct for short wavelengths but not so at long. Planck's initial intuition was to develop an empirically fitting function which would be weighted to the Rayleigh–Jeans Law for long wavelengths and to Wein's equation at short wavelengths, thus matching experimental data. He did exactly that in late 1900 with his equation

$$I_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

However, it took several years of hard work to develop the theoretical basis for his equation.

The total energy radiated by a blackbody per second (its radiated power) per unit area is equal to the area under its blackbody curve. The total energy is arrived at by integrating Planck's equation. Doing so results in

$$Total\ Energy = \sigma T^4$$

in which

$$\sigma = \left[\frac{2\pi^5 k^4}{15c^2h^3}\right] = [5.67\ x\ 10^{-8}]$$

so that

$$Total\ Energy = \sigma T^4 = [5.67\ x\ 10^{-8}] \cdot T^4$$

which is the Stefan-Boltzmann law. This law was developed experimentally in 1879 by Austrian physicist Josef Stefan. In 1884 the same law was derived by Austrian physicist Ludwig Boltzmann from thermodynamic considerations. Since

$$Total\ Energy = \sigma T^4$$

the energy radiated per second per unit area by a blackbody increases rapidly as its temperature raises.

5.1.3 Difference Between the Rayleigh–Jeans Law and Planck's equation

The Rayleigh–Jeans Law

$$I_\lambda(T) = \frac{2c}{\lambda^4} \cdot kT$$

was derived assuming that the energy spectrum of an object's vibrating atoms was continuous. That is, an individual atom could possess any amount of energy.

In contrast, Planck proposed that an atom could only vibrate at specific harmonically related quantum's of energy

$$E_n = \frac{n\,h\,c}{\lambda}$$

with n = 1, 2, 3, etc. Using quantized instead of a continuous energy distribution resulted in the equation

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

instead of

$$I_\lambda(T) = \frac{2c}{\lambda^4} \cdot kT$$

As we have seen, for small values of $hc/\lambda kT$ (long wavelengths and low temperatures)

$$e^{hc/\lambda kT} - 1 \approx \frac{hc}{\lambda kT}$$

reducing Planck's equation to

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \approx \frac{2hc^2}{\lambda^5} \frac{\lambda kT}{hc} = \frac{2c}{\lambda^4} \cdot kT$$

which is the same as Rayleigh–Jeans law

$$I_\lambda(T) = \frac{2c}{\lambda^4} \cdot kT$$

So, Planck's equation is the same as the Rayleigh–Jeans Law at low temperatures and long wavelengths, both going to zero as wavelengths become longer. However, at short wavelengths and high temperatures the Rayleigh–Jeans Law explodes going to infinity because it does not have the compensating exponential term

$$\frac{1}{e^{hc/\lambda kT} - 1}$$

With the exponential term, Planck's equation

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

does go to zero at short wavelengths and high temperatures as it must.

Consequently, by quantizing energy such that

$$E_n = n \cdot \frac{h\,c}{\lambda}$$

Planck came up with the correct equation for blackbody radiation.

Planck received the 1918 Nobel Prize in Physics "in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta".

5.2 The Photoelectric Effect

In 1887 Heinrich Hertz (1857 – 1894) discovered the photoelectric effect. The following year he used a primitive spark-gap transmitter to prove that Maxwell's theory of electromagnetic waves was correct (Figure 7).

The apparatus used by Hertz in discovering the photoelectric effect is illustrated in Figure 8. The equipment consisted of an evacuated glass tube with an electrode at each end. One electrode, the detector in Figure 8, was connected to the positive terminal of a battery with the other electrode, the metal surface, attached to the battery's negative terminal. The object of the experiment was to see if an electrical current could be generated by shining light on one electrode or the other. The theory was that the electric field between the two electrodes, created by the battery, would cause any charged particles eject by the light (Figure 9) to flow from one electrode to the other. The presence of an electrical current, if any, was detected by an ammeter connected in the circuit.

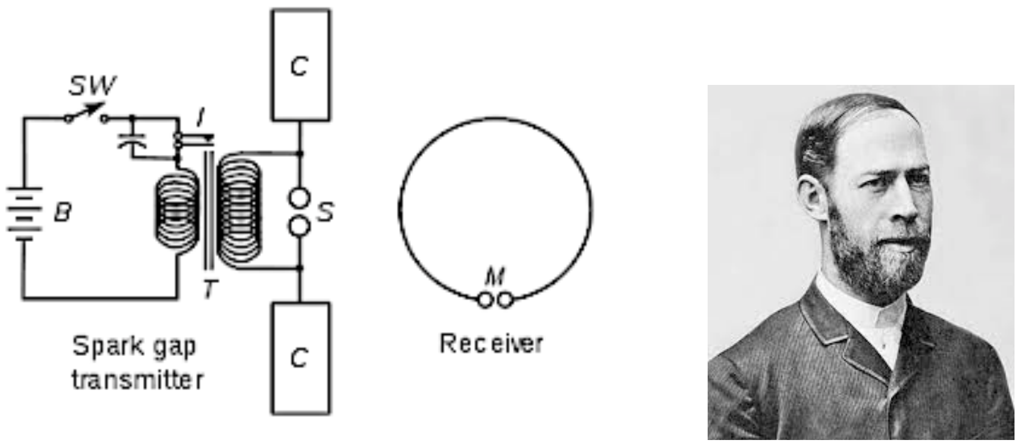


Figure 7 Hertz equipment used to detect electromagnetic waves (source: Wikipedia)

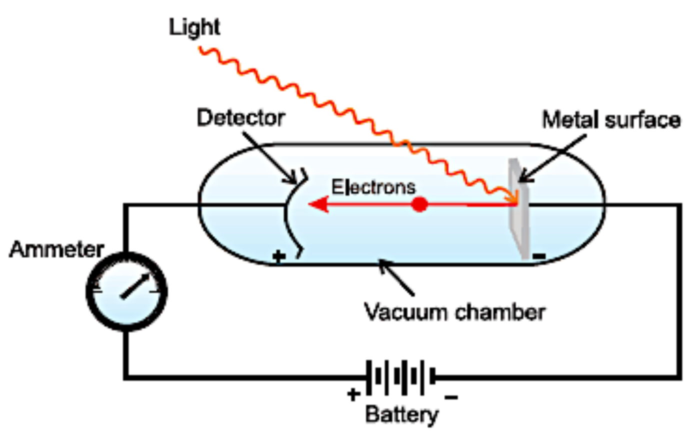


Figure 8 Apparatus used by Hertz to discover the Photoelectric effect
(source: Zap Science)

Initially no results were obtained. But an electrical current suddenly appeared when the metal surface was illuminated with blue light. Charged particles ejected from the metal surface by the incident light flowed through the tube to the detector. At longer wavelengths (yellow through red light) no particles were ejected regardless of light intensity. In 1899 J. J. Thomson confirmed that the ejected particles in the photoelectric experiments were electrons.

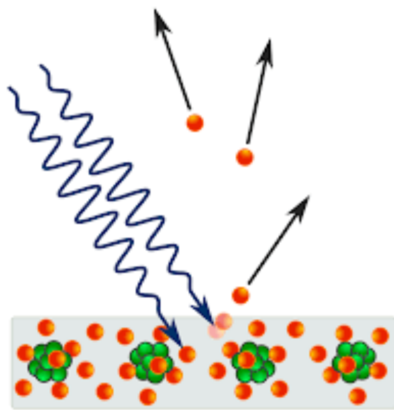


Figure 9 Electrons ejected from metal surface by incident light (source: Wikipedia)

The following observations were made by Hertz and others in the fifteen years or so following the initial experiment.

1. Each type of metal used in constructing the electrodes has its own threshold frequency.

2. No electrons will be emitted if the frequency of the incident light is less than the threshold frequency, regardless of the light's intensity.

3. Electrons will be emitted immediately the moment the metal surface is illuminated with light above the threshold frequency, no matter how low the light intensity.

4. The number of electrons ejected from the metal surface increases with light intensity at frequencies above the threshold frequency.

5. The increase in the number of electrons ejected depends only on light intensity, not how much the frequency of the incident light exceeds the threshold level.

6. The kinetic energy of the ejected electrons depends on the frequency of the incident light but not on the intensity of that light.

7. The kinetic energy of the ejected electrons increases linearly with the frequency of the incident light.

Contemporary physics at the time (classical physics) could not explain these results.

It was believed that any frequency of light with sufficient intensity could eject electrons from a metal surface. However, this disagreed with the experimental results that light had to be above a threshold frequency in order to eject electrons.

What would happen if the light intensity was very weak? According to classical physics an electron would continue absorbing energy until it gained sufficient energy to escape from the metal surface. At very low light intensities it could take an electron hours to absorb enough energy to break free from the metal. But that is not what happened. Electrons were ejected immediately as soon as the incident light exceed the threshold frequency. To push the limits, light intensity was set so low that it should have taken many hours for the photoelectric effect to begin. But even at these very low intensities, the photoelectric effect began immediately as soon as the incident light exceed the threshold level.

Further experiments showed that increasing intensity alone would not eject electrons no matter how high the intensity. The only way that electrons could be ejected was increasing the frequency of the incident light above the threshold level.

The number of electrons ejected increasing with the intensity of light above the threshold level made sense. It was also expected that the number of electrons ejected would increase the higher the incident light frequency was above the threshold level. But this didn't happen either. The experimenters just couldn't get a break. Above the threshold frequency, the number of electrons ejected increased only with light intensity.

It was believed that the kinetic energy of the ejected electrons would increase with both an increase in light intensity as well as frequency. But that didn't. Kinetic energy increased only with the frequency of the incident light above the threshold level, and then it did so linearly with increasing frequency.

In 1905 Einstein (Figure 10) concluded that the problems posed by the photoelectric effect could be resolved by using Planck's quantization of energy. If energy is quantized then light must be quantized as well, in the case of light into small packets of energy which Einstein named photons. A photon is a massless particle that travels at the speed of light. Based on Planck's work, Einstein concluded that the energy of a photon is

$$E = h \cdot f$$

where, as before

E = energy

h = Planck's constant

f = the frequency of light forming the packet of energy which is the photon

Figure 10 Albert Einstein (1879 – 1955) (source: commons.wikipedia.org)

Using his concept of a photon, Einstein explained the photoelectric effect this way.

A certain amount of work ϕ is required to dislodge an electron from a metal surface. The amount of work depends on the type of metal.

When a photon collides with an electron all of the photon's energy $h \cdot f$ is transferred to the electron. An electron will be ejected from a metal surface if the amount of energy absorbed from a photon is more than the metal's work energy ϕ. That is, it will be ejected if

$$h \cdot f \geq \phi$$

At the threshold frequency f_{th}

$$h \cdot f_{th} = \phi$$

Consequently, the frequency f of a photon must be greater than the threshold frequency f_{th} in order for an electron to be ejected. Einstein further assumed, based on experimental evidence, that the energy absorbed by an electron is not cumulative. Either the energy absorbed from a photon is equal to or greater than ϕ, in which case the electron is ejected, or it isn't meaning that the electron is not ejected. If it is not ejected, the energy absorbed from the photon is reradiated by the electron. Because of the reradiation, energy does not accumulate in an electron. Since energy is not cumulative, it doesn't matter what the intensity of the light is (how many photons are present), an electron will not be ejected unless the frequency of the colliding photon is greater than or equal to f_{th} .

Based on this, the number of electrons ejected increases with light intensity at frequencies above the threshold frequency. The higher the light intensity, the more photons at frequencies greater than or equal to f_{th} , and thus the more electrons ejected. Since the threshold frequency is solely responsible for whether an electron is ejected or not, increasing the frequency above threshold has no effect on the number of electrons ejected. An electron will be ejected whenever the frequency of the colliding photon is equal to or greater than the threshold frequency.

The kinetic energy KE of an ejected electron is equal to the difference between the electron's work energy and the energy of the absorbed photon, that is

$$KE = hf - \phi$$

Consequently, the kinetic energy of an ejected electrons depends only on the frequency of the incident light (photons), not on the intensity of that light. Furthermore, the kinetic energy of the ejected electrons increases linearly with the frequency of the incident light as illustrated in Figure 11. Notice in this figure that the kinetic energy of an electron is greater than zero (it is ejected) only at frequencies above the threshold frequency. The work energy ϕ is represented in this figure as a negative energy. Finally, the slope of the linear KE vs frequency curve is simply Planck's constant h.

$$slope = \frac{KE}{f} = h$$

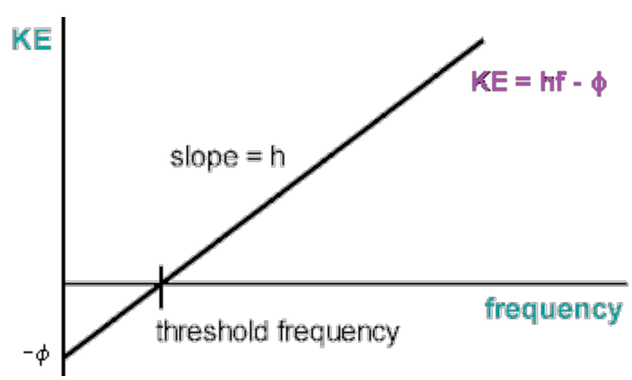


Figure 11 Photoelectric effect kinetic energy (source: toppr)

In 1921 Albert Einstein received the Nobel Prize in Physics "for his services to Theoretical Physics and especially for his discovery of the law of the photoelectric effect".

5.3 Ernest Rutherford's Discovery of the Atomic Nucleus

In 1907 Ernest Rutherford (Figure 12) became professor of physics at Manchester University. The university is located in the city of Manchester, one of the largest cities in North West England. The following year (in 1908) Rutherford was awarded the Nobel Prize in Chemistry for the work that he had done at McGill University (Montreal, Canada) in developing his atomic radioactive disintegration theory.

Figure 12 Physicist Ernest Rutherford (1871 – 1937) (source: Wikipedia)

One of his new students, Ernest Marsden, needed a research project. So Rutherford suggested that they see if any alpha particles would be deflected at large angles (over 90 degrees) when passing through a thin gold foil. From experience α particles passed straight through thin foils, so Rutherford did not expect to see any large angle deflections, but the question had to be investigated.

The experiment he devised is shown in Figure 13. Radioactive radium was used as a source of α particles to bombard a thin foil of gold. The radium was enclosed in a lead box. A small hole in the box formed a beam of α particles aimed at the gold foil. A screen of zinc sulfide was placed around and behind the foil to detect any scattering of α particles. The experiment had to be conducted in the dark so that microscopic flashes of light could be seen each time α particles struck the screen.

As illustrated in Figure 13, most of the α particles passed through the gold foil with little or no deflection striking the screen directly behind the foil. However, occasionally an α particle was deflected at a large angle hitting the screen far from the beam's center line. In a few cases α particles were knocked backwards away from the foil in the direction from which they had come. This was completely unexpected. Rutherford later commented:

"It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

An α particle had to hit a massive object head-on in order to be knocked backwards. What could such an object be? The object had to be small since most of the α particles missed the object passing through the foil without being deflected. The smaller the object was the harder it would be to hit. Since very few α particles were deflected, the particle had to be extremely small.

A gold atom is far more massive than an α particle, massive enough to cause an α particle to be knocked backwards in a head-on collision. To agree with the experimental results most of the atom's mass had to be concentrated in a small object. Rutherford visualized this object residing at the center of the atom. With most of the atom's mass residing in this object, which Rutherford called the nucleus, the remainder of the atom had to be mostly empty space. It was known from Thomson's experiments that the mass of an electron was nearly 2,000 times less than that of a hydrogen atom, and even smaller compared to the mass of a gold atom. (See the chapter on the atom in Related Topics) From this knowledge Rutherford concluded that most of an atom's volume had to consist of electrons orbiting the massive, but very tiny, nucleus at the center of the atom much like the planets of the solar system orbit the Sun. The planets are held in their orbits by the Sun's gravitational force. The electrons had to be held in their orbits around the nucleus by the electrostatic force between the positively charged nucleus and the negative electrons.

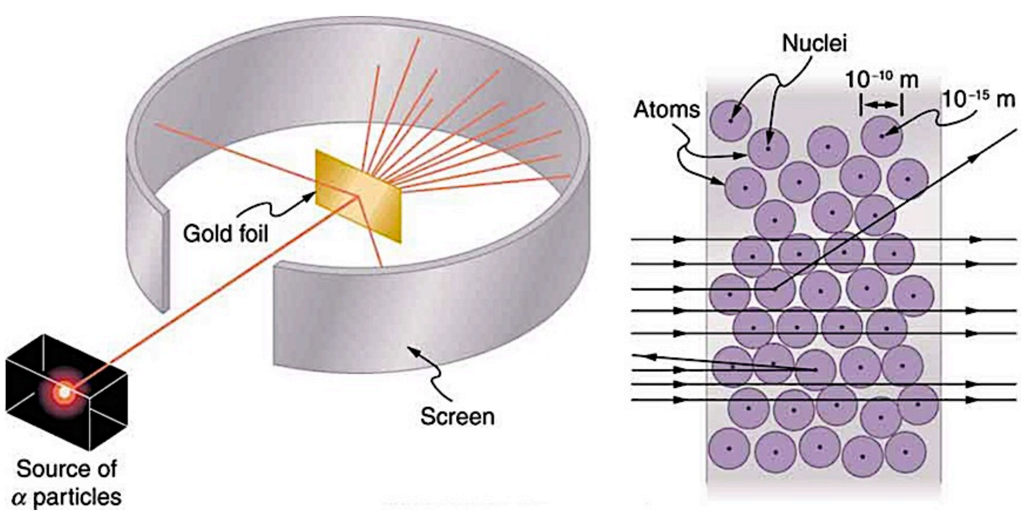


Figure 13 Rutherford's Gold Foil Experiment
(source: https://courses.lumenlearning.com/physics)

Rutherford’s colleagues published the results of the experiment itself in 1909. However, like Thomson before him, Rutherford was reluctant to accept the radical ideas that the experiment seemed to suggest. It took him two more years to convince himself that the experiment was correct and to understand its meaning.

In 1911, Rutherford published his analysis together with a proposed model of the atom (Figure 14). Later the size of the nucleus was determined to be about 10^{-15} m, or 100,000 times smaller than an atom. This implied that the density of the nucleus was huge, on the order of 10^{15} g/cm^3, vastly larger than any macroscopic matter. Also implied by the experiment was the existence of a previously unknown force that could overcome the repulsive Coulomb force to hold the positively charged protons together in the nucleus.

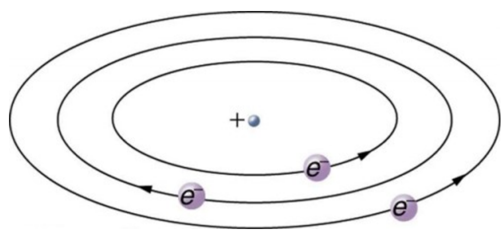


Figure 14 Rutherford’s planetary model of an atom
(source: https://courses.lumenlearning.com/physics

5.4 Niels Bohr’s Atomic Model

Rutherford’s atomic model along with his supporting experimental evidence was one of his great scientific contributions. However, it received little attention beyond Manchester.

In 1913 Danish physicist Niels Bohr (Figure 15) showed the importance of Rutherford’s work. Bohr visited Rutherford’s laboratory the year before and became a faculty member at Manchester from 1914 to 1916.

Rutherford’s atomic model had two serious defects in terms of classical physics.

First, the atom depicted in Rutherford’s model was theoretically unstable. According to Maxwell’s electromagnetic theory, the electrons in Rutherford’s model should accelerate as they orbited the positively charged nucleus causing them to radiate electromagnetic energy. As they lost energy through radiation their orbits should continuously decrease as they spiraled into the nucleus causing the atom to collapse in about 10 picoseconds. According to classical physics, atoms should not exist.

Second, the frequency of the radiated energy would be the same as the electron's orbital frequency. An electron's orbital frequency would continuously increase as it spiraled into the nucleus causing the frequency of its radiated energy to also continuously increase.

These two conclusions completely disagreed with experimental result. First, atoms are stable. They do exist, everywhere. Second, it was known from line spectrums collected over many years that atoms do not radiate energy over a continuous spectrum. Instead, they radiate energy only at discrete frequencies.

Figure 15 Danish physicist Niels Bohr (1885 – 1962) (source: Pinterest)

In 1913 Bohr proposed a new atomic model that provided convincing arguments for the stability of atoms as well as their spectra. Bohr's model was based on Rutherford's planetary scheme. Like Rutherford, Bohr assumed that electrons orbited around the nucleus under the influence of electrostatic attraction from the nucleus. However, Bohr made several assumptions based on Planck's quantization of energy and Einstein's photon theory. Bohr's assumptions violated classical physics but were immensely successful in explaining many properties of the hydrogen atom:

- Instead of orbits at continuously increasing distances from the nucleus, orbital distances were quantized. Only specific orbits at discrete distances from the nucleus were allowed.

- The only allowed orbits were those for which the orbital angular momentum L of an electron was an integer multiple of $h/2\pi$, that is

$$L = n\frac{h}{2\pi} \qquad n = 1, 2, 3, \cdots$$

  where h is Planck's constant (h = $6.62607004 \times 10^{-34}$ m^2 kg / s) and n is called the principal quantum number.

- As long as an electron was in a permitted orbit, it would not radiate electromagnetic energy and thus it would not spiral into the nucleus.

- Emission and absorption of radiated energy could take place only when an electron jumped from one allowed orbit to another. The radiation involved in an electron's transition from an orbit of energy E_n to another orbit with energy E_m was in the form of a photon with an energy of

$$hf = E_m - E_n$$

  f being the frequency of the radiated energy.

Consequently, a hydrogen atom electron absorbed a photon of energy only when it jumped to a higher orbit, the frequency of the absorbed light being:

$$f = \frac{E_m - E_n}{h}$$

Light shining through hydrogen gas develops a black line in its spectrum at a frequency f since the hydrogen gas absorbs light at that frequency. Similarly, an atom emits radiation only when an electron falls back to a lower orbit, emitting a photon of light and creating the emission spectrum shown in Figure 16.

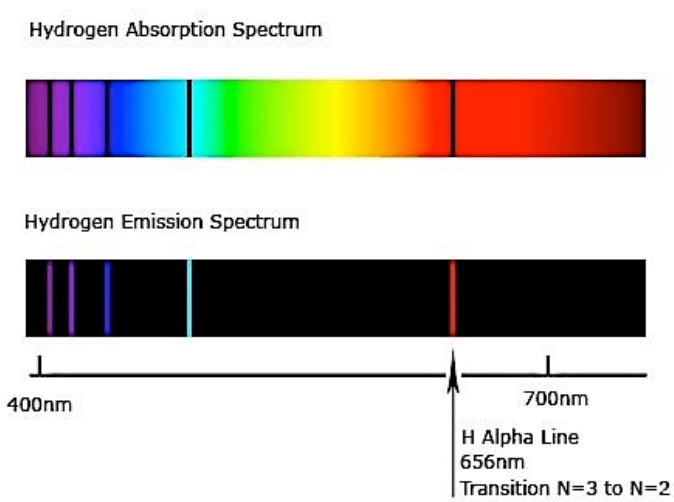


Figure 16
Hydrogen atom
Absorption and
Emission spectrum
(source: Quora)

The energy E_n for the nth orbital of the hydrogen atom, known as Bohr energy, is

$$E_n = -\frac{e^2}{8\pi\varepsilon_0}\frac{1}{r_n} = -\frac{m_e}{2\left(\frac{h}{2\pi}\right)^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{1}{n^2} = -\frac{\mathcal{R}}{n^2}$$

where

n = the nth orbital (prime quantum number)

e = the charge on an electron

ε_0 = electric permittivity of free space = 8.854 x 10^{-12} farad per meter

r_n = the radius of the nth orbital from the nucleus

m_e = mass of an electron

$\mathcal{R}$ = Rydberg constant

$$\mathcal{R} = \frac{m_e}{2\left(\frac{h}{2\pi}\right)^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 = 13.6\ eV$$

Since $E_n = -\mathcal{R}/n^2$ the energy of each orbital of a hydrogen atom is determined by the value of the quantum number n. The negative sign for E_n indicates that the electron is bound to the atom.

The structure of the hydrogen atom energy spectrum E_n is shown in Figures 17 and 18.

n = 1 is defined as the ground state, the lowest energy level (most negative) that an orbital can attain. As n increases the energy level separations decrease rapidly as can be seen in Figure 17. Since n can have any integer value from $n = 1\ to\ n = \infty$, the energy spectrum of the hydrogen atom has an infinite number of discrete energy levels. In the ground state (n = 1) the atom has an energy of

$$E_1 = -\mathcal{R} = -13.6\ eV \qquad and\ a\ radius\ \ a_0$$

The states n = 2, 3, 4, … are referred to as excited states since their energies are greater (less negative) than the ground energy state.

Notice in Figures 17 and 18 that electrons are not restricted to jumping only between adjacent levels. They can in fact jump between any two permitted energy levels. In Figures 17 and 18 the Lyman spectral lines are formed by electrons dropping to the ground state (n = 1) from higher energy levels. The Lyman lines are in the ultraviolet part of the spectrum. The Balmer spectral lines are formed by electrons dropping to the n = 2 energy level. These lines are in the visible part of the spectrum. The Paschen lines (electrons dropping to n = 3) are also in the visible region where as the Brackett and Pfund lines are in the infrared region.

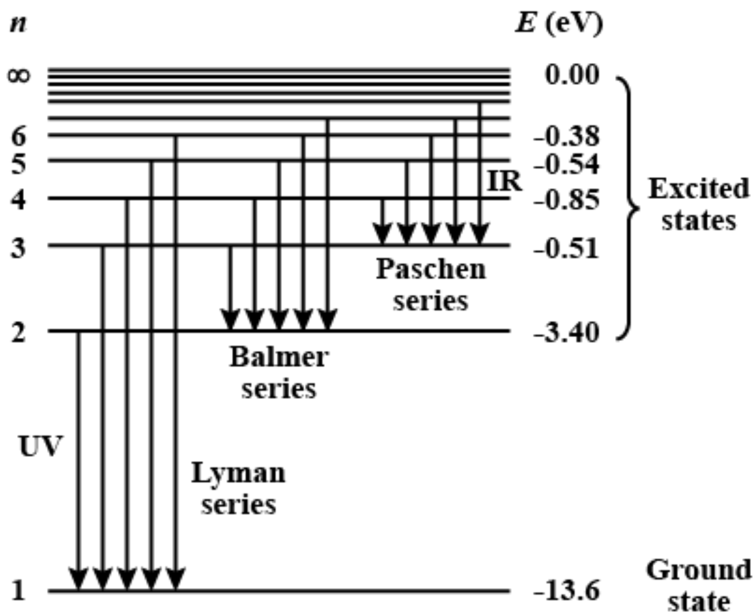


Figure 17
Energy levels for hydrogen atom
(source: Topper)

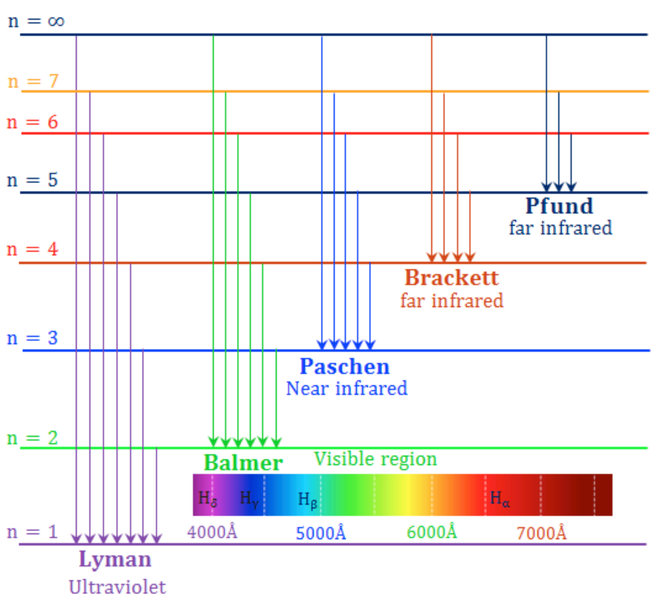


Figure 18
Hydrogen atom
Spectral series
(source: Pinterest)

When the quantum number n is very large $(n \to +\infty)$ the atom's radius r_n will also be very large. However, as n becomes large, the values of its higher energy levels go to zero $(E_n \to 0)$ as illustrated in Figure 17. This means that the proton and the electron, for all practical purposes, are infinitely far apart and hence they are no longer bound together. That is, the atom is ionized with the electron becoming a free particle capable of having any amount of kinetic energy.

In 1922 Bohr received the Nobel Prize in Physics for his foundational contribution to the understanding of atomic structure and quantum theory.

5.5 Arthur Compton's X-ray Scattering Experiment

In his 1923 experiment American physicist Arthur Compton (Figure 19) provided conclusive evidence supporting the particle nature of light. In his experiment, Compton bombarded atoms in a graphite material with a beam of X-ray radiation and measured the amount of X-ray scattering that occurred. In Figure 20, an incident beam of X-ray radiation collides with a graphite atom knocking an electron out of the atom's outer shell and scattering the X-ray radiation. Notice in this figure that the wavelength of the scattered radiation is longer than the incident wavelength. The angle θ is the angle of scattering. Compton found that the wavelength of the scattered X-rays (λ_f) was indeed longer than that of the incident X-rays (λ_i), as illustrated in Figures 20 and 21.

According to classical physics

- The incident and scattered X-ray radiation should have the same wavelength.
- In addition, the degree of scattering should depend on the intensity of the incident radiation.

However, these things did not happen.

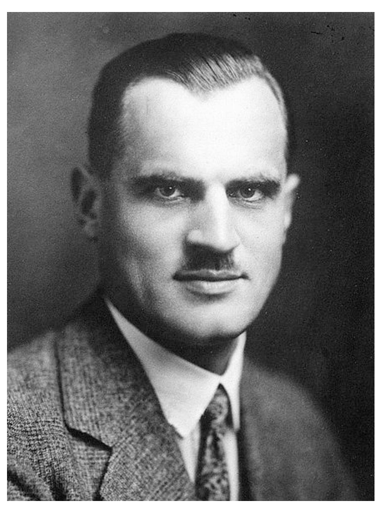


Figure 19 Arthur Compton (1892 – 1962) (source: Wikipedia)

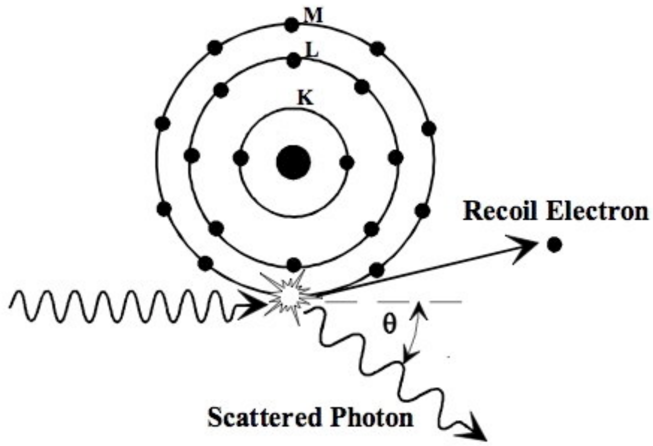


Figure 20 Photon scattering (source: ResearchGate)

Compton's experiment showed that

- The wavelength of the scattered X-ray radiation λ_f increases (became longer) by an amount $\Delta\lambda$, called the wavelength shift, such that $\Delta\lambda = \lambda_f - \lambda_i$
- Furthermore, the wavelength shift $\Delta\lambda$ does not depend on the intensity of the incident radiation but only on the scattering angle θ, as illustrated in Figure 21.

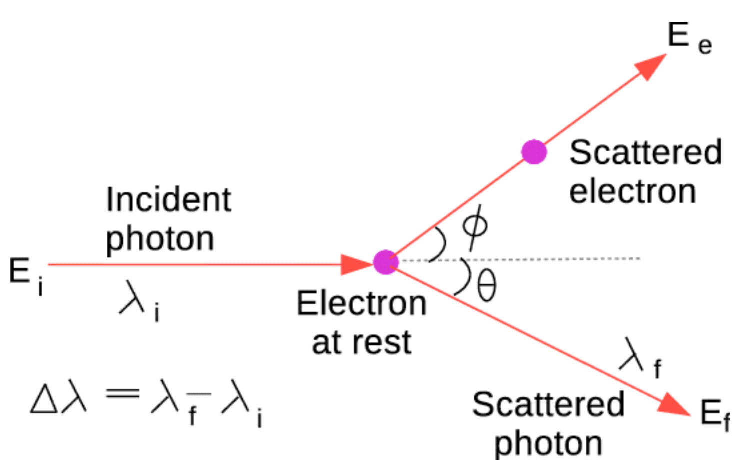


Figure 21 Compton scattering process (source: Imaging H3D, Inc.)

Compton was able to explain his experimental results only by assuming that the incident X-ray radiation was a stream of particles (photons). When a photon collided with an electron it did so elastically in the same way that two billiard balls collide. With this assumption the conservation of energy and momentum laws could be used to analyze the scattering process. Doing so Compton arrived at the following result

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

where

m_e = the mass of an electron ($m_e = 9.10938356 \times 10^{-31}$ kilograms)

c = the speed of light (c = 299 792 458 m / s)

h = Planck's constant ($h = 6.62607004 \times 10^{-34}$ m^2 kg / s)

θ = the scattering angle shown in Figure 21

The term

$$\lambda_e = \frac{h}{m_e c} = 2.426\ x\ 10^{-12} = 2.426\ x\ 10^{-3}\ nano - meters\ (nm)$$

is a constant called the Compton wavelength of an electron.

λ_e is very small compared to the blackbody wavelengths of stars shown in Figure 22. In fact, 100,000 times smaller than the wavelength of ultra-violate light.

Compton's electron wavelength term is derived as follows. Energy

$$E = h\frac{c}{\lambda}$$

Wavelength

$$\lambda = h\frac{c}{E}$$

The energy of an electron with a mass of m_e is

$$E_e = m_e c^2$$

So the electron wavelength defined by Compton is

$$\lambda_e = h\frac{c}{m_e c^2} = \frac{h}{m_e c} = 2.426\ x\ 10^{-3}\ nm$$

Compton's scattering equation then becomes

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_e(1 - \cos\theta)$$

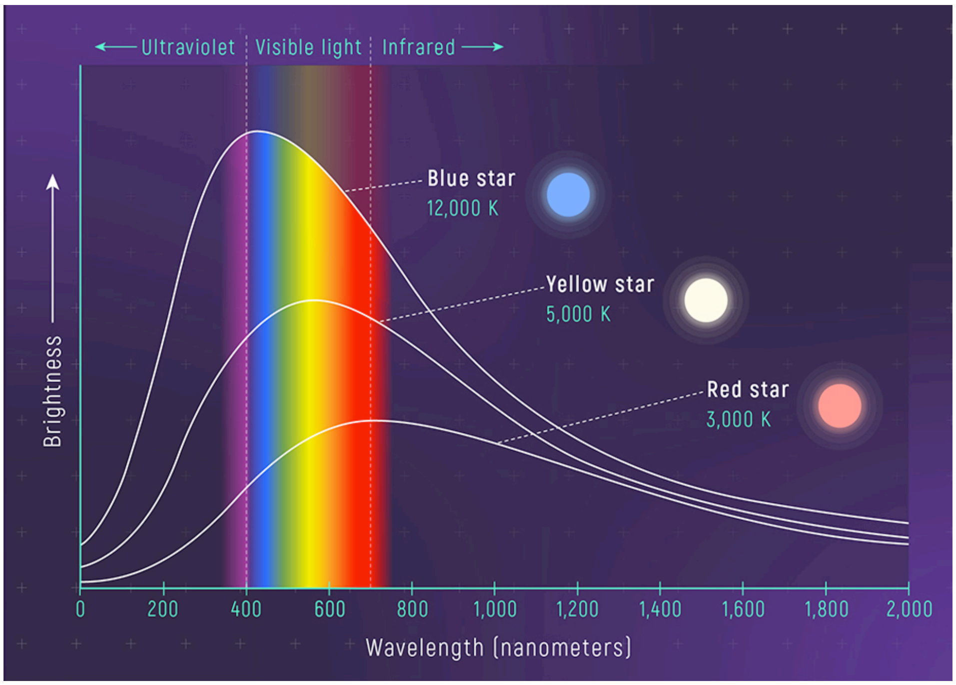


Figure 22 Blackbody spectrum of stars (source: NASA)

The change in photon wavelength $\Delta\lambda$ in Compton's equation is zero if a photon goes straight through a material without being scattered. That is, the scattering angle $\theta = 0$ so $\cos\theta = 1$ and

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_e(1 - \cos\theta) = \lambda_e(1 - 1) = 0$$

If the scattering angle $\theta = 90°$, $\cos\theta = 0$ and

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_e(1 - \cos\theta) = \lambda_e(1 - 0) = \lambda_e$$

In this case the change in photon wavelength after scattering is equal to Compton's electron wavelength λ_e.

Finally, if a photon hits an electron head on it will be knocked straight back in the direction from which it came. In this extreme situation $\theta = 180°$, -1 and

$$\Delta\lambda = \lambda_f - \lambda_i = \lambda_e(1 - \cos\theta) = \lambda_e(1 - (-1)) = 2\lambda_e$$

So the maximum changes in a photon's wavelength occurs when it hits an electron head on and is knocked backwards. For this extreme

$$\Delta\lambda_{max} = 2\lambda_e = 4.852\ x\ 10^{-3}\ nm$$

which is still a very small number. This is the greatest change in wavelength that a photon can experience when colliding with an electron.

From the above examples, the wavelength shift $\Delta\lambda$ does not depend on the intensity or frequency of the incident radiation but only on the scattering angle θ. The wavelength shift $\Delta\lambda$ for red light scattered at an angle θ is the same as that for blue light scattered at the same angle.

A photon looses energy when it collides with an electron. The amount of energy that it looses is

$$\Delta E = E_i - E_f = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f} = hc\frac{\lambda_f - \lambda_i}{\lambda_f\lambda_i} = hc\frac{\Delta\lambda}{\lambda_f\lambda_i} = \frac{hc\frac{h}{m_e c}}{\lambda_f\lambda_i} \cdot (1 - \cos\theta)$$

$$\Delta E = \frac{\frac{h^2 \cdot (1 - \cos\theta)}{m_e}}{\lambda_f\lambda_i} = \frac{h^2 \cdot (1 - \cos\theta)}{\lambda_f\lambda_i m_e}$$

$$\Delta E = \frac{h^2 \cdot (1 - \cos\theta)}{\lambda_f\lambda_i m_e}$$

The change in energy depends not only on the scattering angle θ, but is also inversely proportional to the incident λ_i and scattered λ_f wavelengths. The longer these wavelengths, the more toward the red end of the spectrum, the smaller the change in energy for a given scattering angle θ. Conversely, the change in energy is the greatest for short wavelengths toward the blue end of the spectrum.

A photon's energy does not change if it goes straight through a material without being scattered, that is $\theta = 0$. A photon looses the most energy in a head on collision in which the photon is knocked straight backwards. In this case

$$\Delta E = \frac{h^2 \cdot (1 - \cos\theta)}{\lambda_f\lambda_i m_e} = 2\ \frac{h^2}{\lambda_f\lambda_i m_e}$$

Since the energy of a photon is inversely proportional to its wavelength, according to

$$E = h \cdot \frac{c}{\lambda}$$

a loss in energy forces the photon's wavelength to become longer. That is, the incident light beam down shifts in color following a collision.

A photon of a light beam may be scattered several times as it encounters the atoms of a material as illustrated in Figure 23. A photon down shifts in color each time it is scattered. An X-ray photon may down shift to ultra-violate light, ultra-violate to blue, blue to yellow light, and so on.

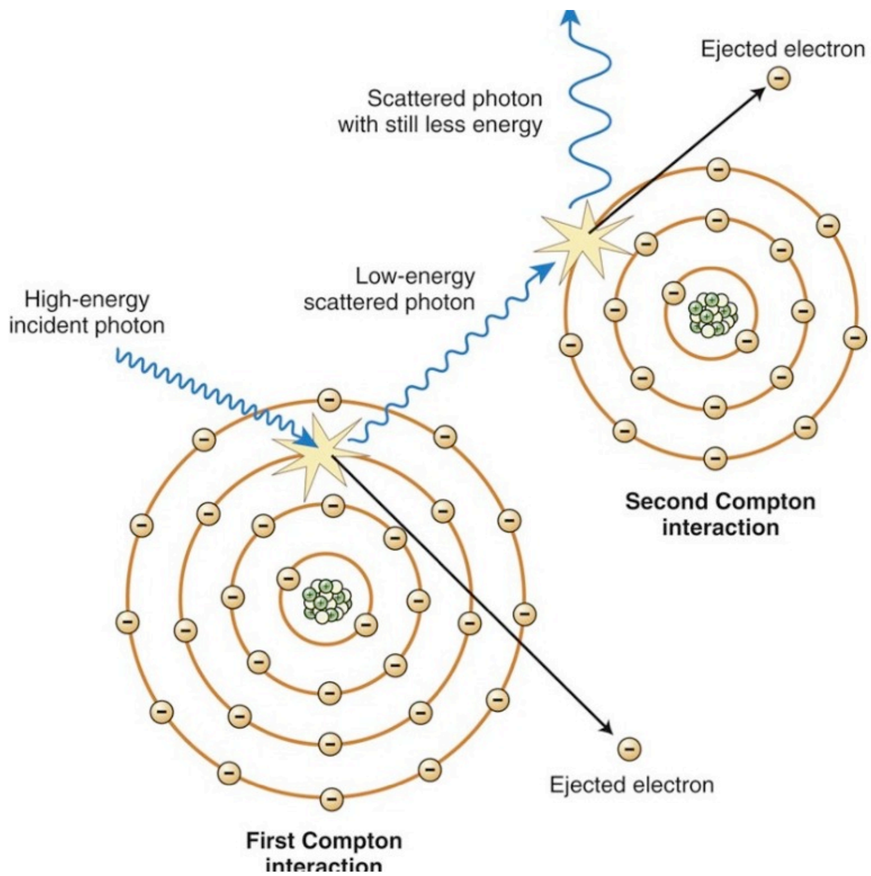


Figure 23 Multiple photon scattering (source: ShowMe)

This exact thing happens in the Sun. Nuclear fusion within the Sun's core (Figure 24) produces huge quantities of gamma radiation as hydrogen is fused into helium. Gamma rays are the highest energy highest frequency form of electromagnetic energy.

Gamma ray photons emanating from the Sun's core are in constant never ending collisions with hydrogen and helium nuclei throughout the Sun's Radiation Zone. The collisions scatter the photons in every possible direction. In addition, each collision transfers small amounts of energy from the energetic photons to the hydrogen and helium nuclei heating the Radiation Zone to its very high temperature.

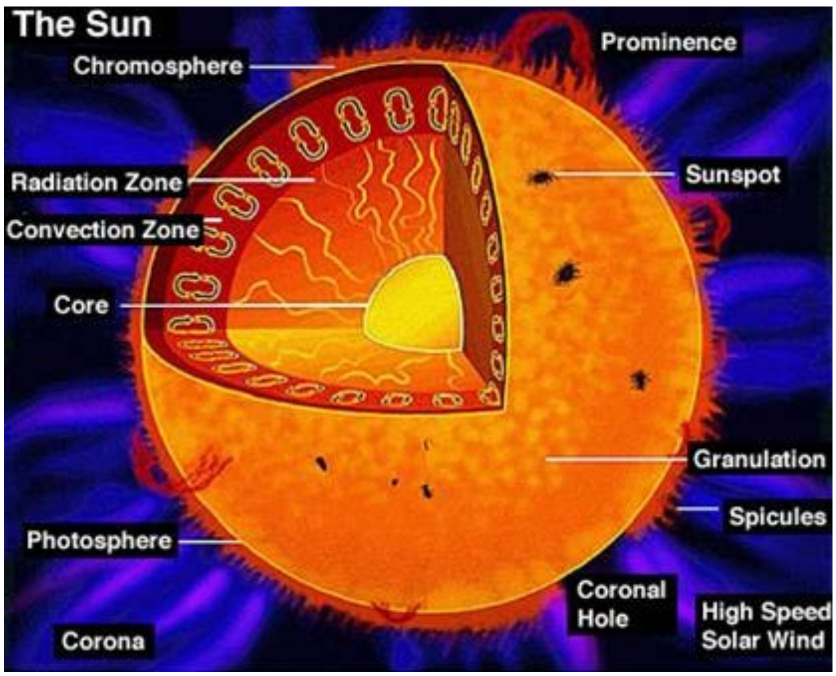


Figure 24 The Sun (credit: NASA's Cosmos – ase.tufts.edu)

Because of its zigzag chaotic path, a photon typically takes around 170,000 years to traverse the Radiation Zone. It then passes quickly through the Convection Layer and takes only 8 minutes to complete its journey from the Sun's Photosphere to Earth. The sunlight that we enjoy today began its journey in the Sun's core shortly after modern man (homo-sapien) first appeared on earth 200,000 years ago. It has been in route to us through all of human history finally arriving today.

Since the energy of a photon is $E = hf$, the frequency of a photon's electromagnetic wave must drop as it looses energy through constant collisions with hydrogen and helium nuclei. Photons which begin as gamma rays in the Sun's core are reduced to X-rays and extreme ultra-violate light by the time they emerge from the Radiation Zone a distance of $0.75R_{\odot}$ from the Sun's center. ($R_{\odot}$ is the Sun's radius). Continued scattering in the

Convection Zone causes many of the photons to down shift further in color producing the spectrum of photons that we recognize as visible light radiating from the Sun.

Figure 23 also illustrates the ionization process. When a photon of extreme ultra-violate light from the Sun collides with an electron in a nitrogen or oxygen atom in Earth's upper atmosphere, the photon is scattered and an electron is knocked out of the atom. In the process a free electron and a nitrogen or oxygen ion is formed creating the Earth's ionosphere as billions of atoms are ionized, illustrated in Figure 25.

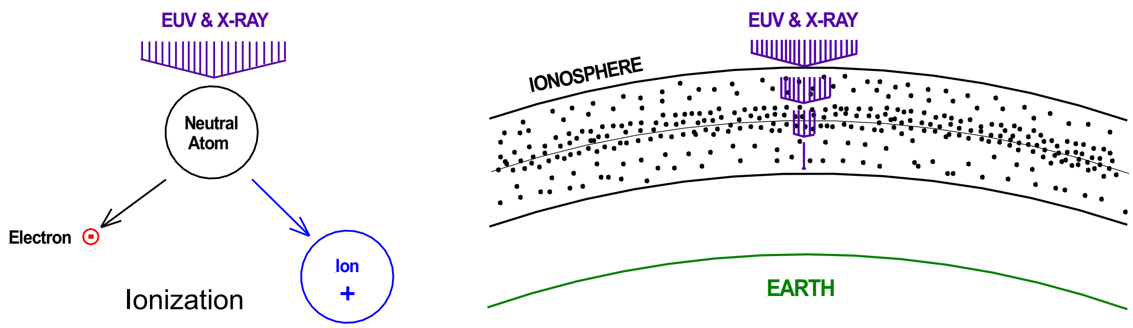


Figure 25 Ionization of Earth's upper atomosphere (source: author)

Compton won the 1927 Nobel Prize in Physics for his 1923 discovery of the Compton effect which demonstrated the particle nature of electromagnetic radiation

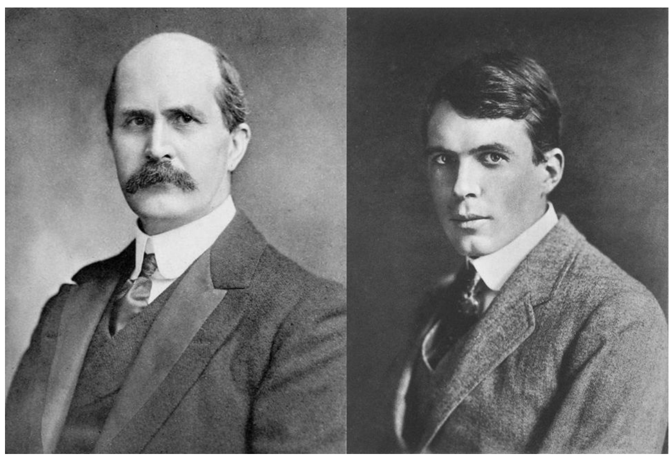
5.6 Bragg Diffraction

The experiments of Young and Foucault demonstrated that at the human level light is a wave. Einstein and Compton, however, showed that microscopically on the scale of atoms light is a particle. These experiments could lead one to believe that light behaves as a wave only in our human macroscopic world, but in reality, at the microscopic level, light is actually a particle. The Bragg father and son team (Figure 26) showed that this is not true. They proved that light can also behave as a wave at the microscopic atomic scale.

In 1913 they discovered that interference patterns were produced when X-rays were passed through crystalline solids such as sodium chloride (Figure 27). The intensity of the interference pattern maximum and minimums depended on both the X-ray wavelength and angle of incidence with the crystalline solid. Light with wavelengths longer than X-rays did not create interference patterns.

The interference patterns were produced by reflection of the X-rays from two or more crystallographic planes as illustrated in Figure 28.

It is important to note that at the atomic scale we can no longer talk about reflection of light in the classical sense. Instead, we must think of photons being absorbed by atoms and then re-emitted in all directions as spherical waves. Absorption of photons cause electrons in an atom to jump to higher energy levels. Eventually they fall back from these higher levels to their original ground states. As electrons fall back they radiate photons in all directions with the same energy (frequency) as the incident photons. The spherical wavefronts produced by the radiated photons interact forming interference patterns.



William Henry Bragg William Lawrence Bragg

Figure 26 Bragg Father and Son (source: physicsworld.com)

The important parameters in the reflection of X-rays from the various crystal lattice planes are

- The distance d between lattice planes,
- Wavelength λ of the incident X-rays which must be comparable in size to the distance d between lattice planes, and
- θ which is the angle of incidence of the incoming X-rays with the horizonal (in Figure 28) planes of the crystal lattice. Since the angle of incidence equals the angle of reflection, the angles shown in Figure 28 are 2θ .

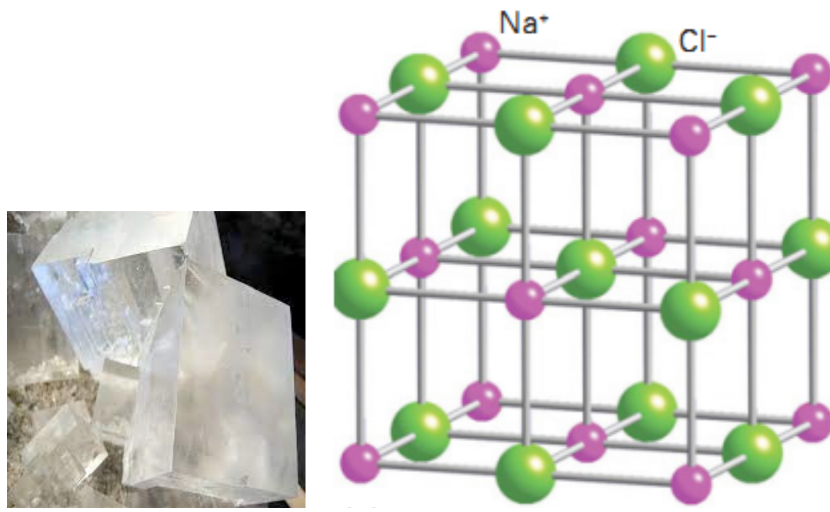


Figure 27 Sodium Chloride NaCl lattice structure (source: Chemistry)

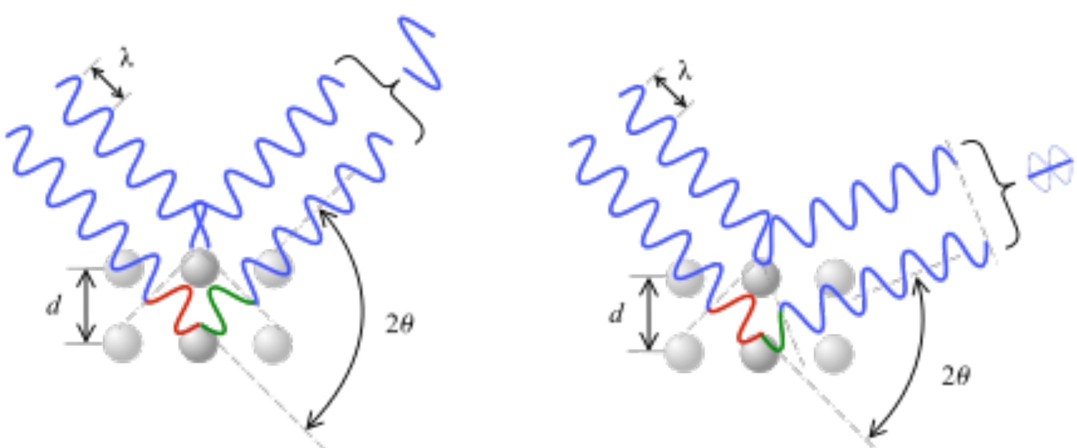


Figure 28 Bragg’s Law (source: Wikipedia)

In Figure 28 the waves on the left interfere constructively while those on the right interfere destructively. The upper wave on the left reflects off the middle atom in the top row of the crystal lattice. The bottom wave reflects off the middle atom in the next lower lattice plane. The distance traveled by the bottom wave is longer than that of the upper wave. The difference in distance traveled is equal to the sum of the red and green partial cycles. The two reflected waves will be in phase and interfere constructively if the difference in distance is an integral multiple n of wavelengths $n\lambda$ where n = 0, 1, 2, 3, …. that is

$$0, \qquad \lambda, \qquad 2\lambda, \qquad 3\lambda, \qquad etc$$

The amplitude of the resulting wave after constructive interference is twice that of the incident waves as illustrated in Figure 28. However, if the path difference is an odd number of half wavelengths $m\lambda/2$ where m = 1, 3, 5, ……

$$\frac{1}{2}\lambda, \qquad \frac{3}{2}\lambda\,, \qquad \frac{5}{2}\lambda, \qquad etc$$

then the two waves will add destructively, completely canceling each other out as occurs with the waves on the right in Figure 28.

Notice on the left side of this figure that the sum of the red and green partial waves is two wavelengths. The red wave is 1 wavelength and the green wave is also 1 wavelength resulting in a total path length difference of two wavelengths

$$difference \ \approx 2\lambda$$

Thus, the waves interfere constructively producing a resulting wave with an amplitude twice that of the original waves (a Bragg maximum).

However, on the right side of the figure the sum of the red and green partial waves is less than 2 wavelengths. The red wave is 1 wavelength long but the green wave is only a half wavelength, resulting in a total path length difference of one and a half wavelengths

$$difference \ \approx \frac{3}{2}\lambda$$

Consequently, the waves on the right destructively interfere completely cancelling one another out (a Bragg minimum).

Of the three parameters $d,\ \lambda,\ and\ \theta$, the distance d between lattice planes is fixed by the type of crystal. The wavelength λ is determined by the frequency of X-rays irradiating the crystal. Consequently, the only variable is the angle of X-ray incidence θ with the crystal. The Bragg team showed that constructive interference occurred when

$$2d \cdot \sin\theta = n\lambda$$

where n must be an integer multiple such that n = 1, 2, 3, … etc.

The Bragg experiment showed that on a microscopic scale light behaves as a wave when interacting with a crystal lattice. This was an extremely important result. It showed that light acts like a wave both on the human scale as well as at the microscopic level of atoms. X-ray radiation incident on an atomic crystal lattice produced interference patterns the same as those produced by Young's double slit experiment. Consequently, at the microscopic level light is **both** a wave and a particle. The way that it behaves (as a wave or as a particle) depending on the situation.

The British farther and son team shared the 1915 Nobel Prize in Physics for their X-ray analysis of crystal structures.

5.7 De Broglie Hypothesis

In 1923 French physicist Louis De Broglie (Figure 29) proposed that not only do light waves exhibit particle behavior, but conversely material particles display wave-like behavior. Material particles include electrons, atoms, molecules, and every material body in the universe. This concept, which forms a central part of quantum mechanics, was confirmed experimentally by American physicists Davisson and Germer in 1927 thus establishing the wave-particle duality.

Figure 29 De Broglie (1892 – 1987) (source: Wikipedia)

The wavelength of a particle is defined by De Broglie's equation

$$\lambda = \frac{h}{p}$$

where

h = Planck's constant $(6.626\ x\ 10^{-34}\ m^2 \cdot kg/s)$, and

p = the momentum of a particle

This equation can be derived as follows

In classical physics, the momentum of a particle equals its mass m times its velocity v, that is

$$p = m \cdot v$$

Using Einstein's equation, energy is equal to

$$E = mc^2$$

where c is the speed of light. So

$$m = \frac{E}{c^2}$$

Momentum then equals

$$p = mv = \frac{E}{c^2} \cdot v$$

For a photon $v = c$, so

$$p_{photon} = mv = \frac{E}{c^2} \cdot v = \frac{Ec}{c^2} = \frac{E}{c}$$

Consequently, momentum of a photon is

$$p_{photon} = \frac{E}{c}$$

The energy of a photon is equal to

$$E_{photon} = hf = h\frac{c}{\lambda}$$

Leading to

$$p_{photon} = \frac{E}{c} = \frac{hc}{\lambda} \cdot \frac{1}{c} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p_{photon}}$$

and De Broglie’s more general equation

$$\lambda = \frac{h}{p}$$

For particles which have a non-zero mass, that is particles other than photons, De Broglie’s equation becomes

$$\lambda = \frac{h}{p} = \frac{h}{m \cdot v}$$

This equation shows that the wavelength of an object gets smaller the more massive it is and the faster it is moving.

For example, the wavelength of a person with a mass of 75 kg (165 lbs.) jogging at $8\ km/h \approx 2.2 m/s$ (about 5 mph) is

$$\lambda = \frac{6.626\ x\ 10^{-34}}{75\ x\ 2.2} = 4.016\ x\ 10^{-34}\ m$$

This wavelength is 7 billion, billion times smaller than the radius of an electron which is about $2.8\ x\ 10^{-15}\ m$. The wavelength of a person is so incredibly small that it is undetectable, but it is still there!

De Broglie’s hypothesis, which is now fact, provides insight into Bohr’s atomic model, specifically the requirement that an electron can only occupy specific quantized orbits. In Figure 30 an electron orbits the nucleus at a radius of r (the dashed circle). Since an electron has particle wave duality, the circumference of the electron’s orbital path must be an exact integer multiple of its wavelength (red curve) as shown in Figure 30. Thus a whole number n of wavelengths must fit exactly around an orbital path such that

$$n\lambda = 2\pi \cdot r$$

$2\pi \cdot r$ being the circumference of the orbital path.

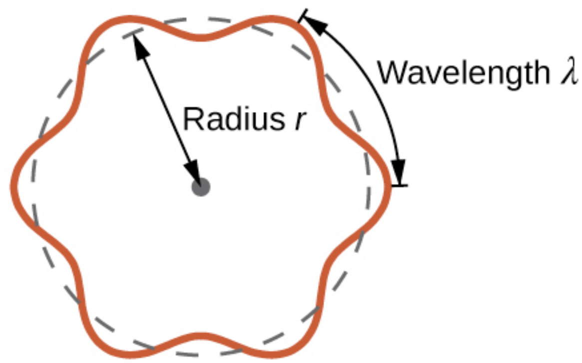


Figure 30 Orbital constraint of an electron (source: chem.libretexts.org)

The orbit shown in Figure 31 is not allowed since an integer number of wavelengths does not fit around the orbit's circular path.

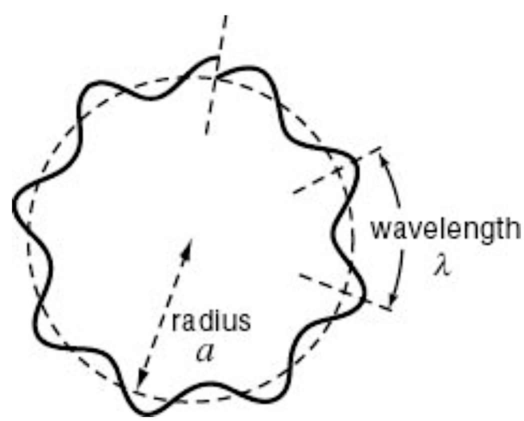


Figure 31 Unallowed orbit (source: socratic)

In Bohr's model of the atom, the only allowed orbits are those for which the orbital angular momentum L of an electron is an integer multiple of $h/2\pi$, that is

$$L = n\frac{h}{2\pi} \qquad n = 1, 2, 3, \cdots$$

The requirement that an integer number of wavelengths must fit around an electron's orbit requires that

$$n \cdot \lambda = 2\pi \cdot r$$

And finally, de Broglie's equation states that

$$\lambda = \frac{h}{p} = \frac{h}{m \cdot v}$$

Putting these together gives

$$n \cdot \lambda = \frac{n \cdot h}{m \cdot v} = 2\pi \cdot r$$

$$\frac{n \cdot h}{2\pi} = mvr$$

$where\ mvr = angular\ momentum\ L$ which yields Bohr's orbital angular momentum L equation

$$L = n\frac{h}{2\pi}$$

If an electron drops to a lower energy level,

1. Its new orbit has a smaller radius r which means that fewer full wavelengths can fit in the new orbit.

2. The energy difference between two allowed orbits

$$\Delta E = E_m - E_n = hf = \frac{hc}{\lambda}$$

These two requirements means that orbital distances are quantized and that only specific orbits at discrete distances from the nucleus are allowed. In addition, the lowest allowed energy level is always more than zero since a full number of wavelengths must fit in each orbital shell, including the lowest energy level shell.

De Broglie won the 1929 Nobel Prize for Physics after the wave-like behavior of matter was experimentally demonstrated in 1927.

5.8 Conclusion

Light is both a wave and a particle.

At microscopic levels light interacts with crystalline structures reflecting as waves from the crystal atoms forming interference patterns in the process. When colliding with electrons, atoms, etc., light acts like particles (photons) with zero mass that travel at the speed of light. The energy of a photon is

$$E = h \cdot f = h\frac{c}{\lambda}$$

where,

E = energy

c = the speed of light

f = frequency

λ = wavelength

h = Planck's constant (h = 6.62607004 × 10^{-34} m^2 kg / s)

Photons also have momentum p equal to

$$p = \frac{h}{\lambda}$$

Because they have momentum, photons exert a force equal to

$$\frac{dp}{dt} = h\,\frac{d(1/\lambda)}{dt}$$

on the objects that they collide with. A photon looses energy

$$E = h \cdot f = h\frac{c}{\lambda}$$

during a collision causing an increase in its wavelength λ (its wavelength becomes longer) and a drop in its frequency f. Consequently, a photon down shifts in color following each collision. A photon that begins as an X-ray may down shift to ultra-violate light, then to blue light, followed by yellow light, and so on as the result of multiple collisions.

At the human macroscopic scale, light behaves primarily as waves that interfere with one another creating interference patterns, as well as reflecting, refracting, and diffracting as they encounter different materials (media). Light also acts as waves when propagating from one place to another.

The way light behaves (as a wave or a particle) depends on the situation. Some phenomena can only be explained by the particle nature of light. Other can only be explained by light's wave characteristics.

Consequently, light is both a wave and a particle.

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